Reprints from the Early Days of Information Sciences

Paul Ehrenfest – Remarks on Algebra of Logic and Switching Theory

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Reprints from the Early Days of Information Sciences
Paul Ehrenfest – Remarks on Algebra of Logic and Switching Theory
Paul Ehrenfest - Remarks

on

the Algebra of Logic and Switching Theory

2010
Editors’ Notice

This publication has been written and edited by Radomir S. Stanković and Jaakko T. Astola.
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Historical studies about a scientific discipline are a sign of its maturity. When properly understood and carried out, this kind of study is more than an enumeration of facts or giving credit to particular important researchers. It is more a way of discovering and tracing the ways of thinking that have led to important discoveries. In this respect, it is interesting and also important to recall publications where some important concepts, theories, methods, and algorithms were introduced for the first time.

In every branch of science there are some important results published in national or local journals or other publications that have not been widely distributed for different reasons, due to which they often remain unknown to the research community and therefore are rarely referenced. Sometimes the importance of such discoveries is overlooked or underestimated even by the inventors themselves. Such inventions are often re-discovered much later, but their initial sources may remain almost forgotten, and mostly remain sporadically recalled and mentioned within quite limited circles of experts. This is especially often the case with publications in languages other than the English language which is presently the most commonly used language in the scientific world.

This series of publications is aimed at reprinting and, when appropriate, also translating some less known or almost forgotten, but important publications, where some concepts, methods or algorithms were discussed for the first time or introduced independently of other related works.

Another aim of the Reprints is to collect and present in the same place the publications on certain particular subjects of important scholars whose scientific work is signified by contributions to different areas of science.

R.S. Stanković, J.T. Astola
Paul Ehrenfest - Remarks

on

the Algebra of Logic and Switching Theory
Acknowledgments

The Editors are grateful to Mrs. Marju Taavetti, the Librarian of the Library of the Tampere University of Technology, and Mrs. Pirkko Ruotsalainen, the Development Manager of the Department of Signal Processing, Tampere University of Technology, for their help in collecting the relevant literature.

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The Remarks of Paul Ehrenfest on Algebra of Logic

Abstract

The present issue of the Reprints from the Early Days of Information Sciences discusses the remarks of Paul Ehrenfest on the applications of the Algebra of Logic in the design of logic networks. The remarks were made in a review of the Russian edition of the book The Algebra of Logic by Louis Couturat that was published in 1910. This issue contains reprints of the review by Ehrenfest and presents a translation of the review from Russian into English. We believe that this is the first translation of the complete text of this review into English and its first reprint. The remarks by Ehrenfest were mentioned in the reviews of several other publications in the field of logic design. These reviews are also reprinted.
Notice
This book contains several reprints of pages from articles or reviews of articles and books where the remarks of Paul Ehrenfest about applicability of algebra of logic in the design of logic networks were mentioned. These articles and reviews were written by eminent scholars in this field and confirm their knowledge of this work. We did not want to rephrase or rewrite their original statements, since we believe that the way they were presented originally has a particular value for the reader.

We kindly ask for these reprints not to be considered simply as graphic illustrations from previous publications, but to be read as part of the presentation in this book.

English, Serbian Latin, Serbian Cyrillic, Finnish, Russian, Japanese, Armenian, German, Castilian, Georgian, Hungarian, Bask, Estonian, Sami
1 Remarks on the Origins of Switching Theory

Logic networks and many sophisticated techniques for designing them antedate digital computers by many years. The initial applications were in the design of telephone central office equipment. The key concept, which transformed the design process from an art or skills based on the experience of the designers into a science, was the idea of describing both the functions performed and the circuits themselves in terms of Boolean algebra. This observation and related subsequent derivations led to Switching theory as the mathematic foundations for the design of logic networks.

As is is usually the case in engineering and science, a new area or a sub-discipline starts developing by solving first some particular task, with the solution derived based on previous experiences and skills of individuals. If the task is important and the solution efficient and useful, whatever criteria of efficiency and usefulness are, demands for repeated solutions of the same or similar tasks soon arise. Then attempts towards the automatization of the related method or the procedure are naturally made. This necessary requires a formal description of both the problem and the method used to solve it, which requires introduction of certain notions and definitions and leads to the establishing of basic theoretical foundations. Improving performances of solutions and increasing complexity of systems where the task is enrolled, are next to be considered. When the complexity of the system and, therefore, the task, reaches certain level after which it becomes unsolvable by hand and it is hard to produce a solution based just on the experience and skills from practice, some underlying theory is required. Depending on the importance of the problem, formulating such a theory is considered by many scholars in different parts of the world. They are working at about the same time or even simultaneously, however, independently and without knowledge of the work of others. Clearly, researchers might become aware of the related work of others after publication of some results and achievements. After that, authors start referring to the related works of others, as well as try to put their results in a wider context and establish links to the existing related theories.

The development of Switching theory is a typical example of such a scenario of scientific development. In late nineteen thirties, contact and relay networks were widely used in various telecommunication and control systems. The design of these networks was a challenging task requiring a lot of engineering experience and skills. Many researchers had searched for an underlying theory that will enable automatization of the design of
such networks, their simplification, and optimization with respect to various criteria. These research efforts lead to establishing Switching theory as mathematical foundations for Logic design involving Boolean algebra as its central part.

Claude Elwood Shannon presented the idea of using Boolean algebra as a kernel of Switching Theory in his Master Thesis defended in 1938 at the Massachusetts Institute of Technology (Fig. 1). In the Thesis, Shannon provided the following references (in the original formulation as in the thesis)

1. ”A complete bibliography of the literature of symbolic logic”, in *Journal of Symbolic Logic*, Vol. 1, No. 4, December 1936.


5. George Boole, *Finite Differences*, G.E. Strechert & Co., Chap. X.


This thesis is estimated by some scholars as the most frequently referenced master thesis of the 20th century. The main contributions were published in two related papers by C.E. Shannon [26], [27], see Fig. 2, Fig. 3, and Fig. 4. In these publications, Shannon used logic expressions in Boolean algebra to describe and simplify logic networks. Further publications by Shannon in these areas include [23], [24], [28].

In [27], there are 11 references including [23], [26], the book by L.Coutura (item 2 above), and the following references presented here again in the same formulation as in the original paper by Shannon


A SYMBOLIC ANALYSIS
OF
RELAY AND SWITCHING CIRCUITS

by

Claude Elwood Shannon
B.S., University of Michigan
1936

Submitted in Partial Fulfillment of the
Requirements for the Degree of
MASTER OF SCIENCE
from the
Massachusetts Institute of Technology
1940

Signature of Author Claude C. Shannon
Department of Electrical Engineering, August 10, 1937

Signature of Professor
in Charge of Research Frank L. Hitchcock

Signature of Chairman of Department
Committee on Graduate Students Edward E. Woodward

Figure 1: The first page of the MSc. thesis by C.E. Shannon in 1938.
I. Introduction

In the control and protective circuits of complex electrical systems it is frequently necessary to make intricate interconnections of relay contacts and switches. Examples of these circuits occur in automatic telephone exchanges, industrial motor-control equipment, and in almost any circuits designed to perform complex operations automatically. In this paper a mathematical analysis of certain of the properties of such networks will be made. Particular attention will be given to the problem of network synthesis. Given certain characteristics, it is required to find a circuit incorporating these characteristics. The solution of this type of problem is not unique and methods of finding those particular circuits requiring the least number of relay contacts and switch blades will be studied. Methods will also be described for finding any number of circuits equivalent to a given circuit in all operating characteristics. It will be shown that several of the well-known theorems on impedance networks have roughly analogous theorems in relay circuits. Notable among these are the delta-wye and star-mesh transformations, and the duality theorem.

The method of attack on these problems may be described briefly as follows: any circuit is represented by a set of equations, the terms of the equations corresponding to the various relays and switches in the circuit. A calculus is developed for manipulating these equations by simple mathematical processes, most of which are similar to ordinary algebraic algorithms. This calculus is shown to be exactly analogous to the calculus of propositions used in the symbolic study of logic. For the synthesis problem the desired characteristics are first written as a system of equations, and the equations are then manipulated into the form representing the simplest circuit. The circuit may then be immediately drawn from the equations. By this method it is always possible to find the simplest circuit containing only series and parallel connections, and in some cases the simplest circuit containing any type of connection.

Our notation is taken chiefly from symbolic logic. Of the many systems in common use we have chosen the one which seems simplest and most suggestive for our interpretation. Some of our phraseology, such as node, mesh, delta, wye, etc., is borrowed from ordinary network theory for simple concepts in switching circuits.

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A Symbolic Analysis of Relay and Switching Circuits

Claude E. Shannon


Claude E. Shannon is a research assistant in the department of electrical engineering at Massachusetts Institute of Technology, Cambridge. This paper is an abstract of a thesis presented at MIT for the degree of master of science. The author is indebted to Dr. F. L. Hitchcock, Dr. Vannevar Bush, and Dr. S. H. Caldwell, all of MIT, for helpful encouragement and criticism.

Figure 2: The first page of the paper by Shannon [26].
Figure 3: The cover page of the *Bell System J.* where the paper [27] is published.
The Synthesis of Two-Terminal Switching Circuits

By CLAUDE. E. SHANNON

PART I: GENERAL THEORY

1. Introduction

THE theory of switching circuits may be divided into two major divisions, analysis and synthesis. The problem of analysis, determining the manner of operation of a given switching circuit, is comparatively simple. The inverse problem of finding a circuit satisfying certain given operating conditions, and in particular the best circuit is, in general, more difficult and more important from the practical standpoint. A basic part of the general synthesis problem is the design of a two-terminal network with given operating characteristics, and we shall consider some aspects of this problem.

Switching circuits can be studied by means of Boolean Algebra. This is a branch of mathematics that was first investigated by George Boole in connection with the study of logic, and has since been applied in various other fields, such as an axiomatic formulation of Biology, the study of neural networks in the nervous system, the analysis of insurance policies, probability and set theory, etc.

Perhaps the simplest interpretation of Boolean Algebra and the one closest to the application to switching circuits is in terms of propositions. A letter X, say, in the algebra corresponds to a logical proposition. The sum of two letters \(X + Y\) represents the proposition "\(X \text{ or } Y\)" and the product \(XY\) represents the proposition "\(X \text{ and } Y\)". The symbol \(X'\) is used to represent the negation of proposition \(X\), i.e. the proposition "not \(X\)". The constants 1 and 0 represent truth and falsity respectively. Thus \(X + Y = 1\) means \(X\) or \(Y\) is true, while \(X + YZ' = 0\) means \(X\) or \((Y \text{ and the contradiction of } Z)\) is false.

The interpretation of Boolean Algebra in terms of switching circuits is very similar. The symbol \(X\) in the algebra is interpreted to mean a make (front) contact on a relay or switch. The negation of \(X\), written \(X'\), represents a break (back) contact on the relay or switch. The constants 0 and 1 represent closed and open circuits respectively and the combining operations of addition and multiplication correspond to series and parallel connections of the switching elements involved. These conventions are shown in Fig. 1. With this identification it is possible to write an algebraic

Figure 4: The first page of the paper by Shannon [27].

In Japan, the same problem was studied by Akira Nakashima who published in 1935 his research results in [14], [15], [16] (Fig. 5, Fig. 6). Nakashima had an approach opposite to that used by Shannon, he analyzed a large number of different relay networks and devised an underlying theory. With the help of his associate, Masao Hanzawa, Nakashima formulated a theory that can be viewed as a kernel of Switching Theory. In the first seven condensed English translations of his papers, Nakashima does not provide references, except the second paper [18] where there is a reference to his first paper [14]. In 1941, Nakashima and Hanzawa [19], realized the relationship and strong coincidence of their theory with the work of G.J. Boole and E. Schröder and put references to their work [4], [25]


2. E. Schröder, *Vorlesungen über die Algebra der Logik*, Band 1, 1890.

In [21], there is the reference to the work of B.A. Bernstein as follows

B.A. Bernstein, ”Postulate for Boolean algebra involving the operation of complete disjunction”, *Annals of Mathematics*, April 1936.

The following paper in Japanese,


for which there is no English translation, is a short tutorial in which Nakashima presented basic postulates and theorems in the Boolean algebra (Fig. 7).

The first reference in this paper is in Japanese, and other three are


第 三 篇 繰電器の時間的作動特性（続）

現の前後経及の著者に主計は繰電器の受入力の制限を生じるから発電される
作業力の時間的変動特性に及ぼす影響に就て論じたが、作業力の量を大
さるとつぶくものは絶対的のものではなく、常に繰電器電流の作業に対して示され
られた作業力に対する相対的のものである。又著者に於て既述に於て示した電
源電流の作業時間は単に作業開始時間のみを意味するもので、一般に発の作
動時間の一部を表してあるに過ぎない。郎も今質の作業時間 t の内容を考へ
て見ると

\[ t = t_1 + t_2 + t_3 \]

発に

\[ t_1 = \text{作業開始時間} \]

\[ t_2 = \text{発動の "速 効" の時間} \]

と表され、先に論じたものは（37）式中の t_1 のみである。t_1 の後つに於て
発電に依る線路の吸引力は機械的螺旋の作業力に打って発電装置を起
始するが、発の機械上定まる慣性の第に直ちには一定速度の運動を起す、又
発の運動速度は作業力の各発電位に於ける值の測定を受け従って所定の作
動時間まで発電するには減る時間 t_2 が必要である。普通に t_1 は此の二者の和
であるが、若し発電の遮り（bouncing 或は chattering）が起こる時は此の遮電時
間 t_2 をも考慮に入れなければならない。

以上の如く時間的作動特性を支配する因子としては前後に引続いて矢之等の
作業力及び螺旋に関するものを考えるべきである。従来下述本論では之等の
説を少しあって見たい。

第 四 章 對抗勢力に関する因子の影響

341 對抗勢力の強さと静的誘化

機械的螺旋機構を持つ繰電器の抵抗勢力に就て、考えて見る。郎も以上の

a. Canti-lever 式螺旋の発する抵抗力

Figure 5: The first page of the paper by Akira Nakashima in Nichiden Geppo
THE THEORY OF RELAY CIRCUIT COMPOSITION

Akira Nakashima, Member
(Nippon Electric Co., Ltd., Tokyo)

CONTENT.
I. General Essence of Relay Circuit.
II. On Action Element.
III. On Contact Points.
IV. Considerations Regarding Simple Partial Path.
V. Considerations Regarding Complex Partial Path.
VI. Considerations Regarding Energy Transmitting Path.
VII. Time Action Forms of Relay and Their Objects.
VIII. Some of Fundamental Types of Relay Circuit.
IX. Conclusion.

SYNOPSIS.

This is a general discussion of and a systematic consideration on the composition of so-called relay circuit system, which has made surprising advancement in recent time in connection with automatic telephone exchange, remote control systems, etc.

It shows the fundamental idea and characteristics of the relay circuit, some of the interesting properties and theorems regarding the dynamic geometrical character, and analytical treatises of simple cases, forms of relays, and then some of the fundamental systems of relay circuit composition.

It is noted that transient phenomena which arise inevitably in the relay circuit are not, however, included in this discussion.

I. GENERAL ESSENCE OF RELAY CIRCUIT.

1.1. Fundamental idea and definitions.

The relay circuit now in use are many in kinds and complex in variation. However, under general survey, the following definitions may briefly be given:

Relay Circuit is a method in which it becomes a mediator between some given phenomena and the corresponding desired phenomena, and by the use of relays as its composite elements, the occurrence of the former realizes the latter.

Next, taking these relays as its composite elements in broad sense:

The relays may be defined as an element that determines, by presence or absence of its receiving energy, whether another energy is transmitted or not. In regard to energy the former is called controlling and the latter controlled energy. They are, however, termed merely with respect to one certain relay, and one energy may become sometimes the former and other times the latter. Thus, from this definition we find that the relay consists, in one part, of receiving controlling energy to determine its action, and in another part, of controlling directly the transmission of controlled energy. The former may be called acting element and the latter contact point. Contact point may be said as a composite element of transmitting path of controlled energy, which is controlled at its point. It is also made of a number of contact elements that have definite meaning in regard to mechanical contact point.

Accordingly, in the relay circuit, the energy transmitting path is made in general of energy source, acting elements, contact points, and of path element which is a part of the path that connects all of these. The path element containing no impedance against controlled energy is called simple...
Figure 7: The page 4 of the paper by Nakashima published in March 1941.
The paper

is the speech delivered by Akira Nakashima at the general assembly of the IECEJ on 26th April 1941. It covers his major research results. At the third page of this paper, there is a table with basic postulates and theorems in the Boolean algebra. Besides references as in the above paper, Nakashima has mentioned the Journal of Symbolic Logic, 1936, and the book by Couturat (Fig. 8).

For more details on the work of Nakashima and a list of publications, see [32], [34], [35].

In The Soviet Union, early research on this subject was also done in the late thirties, resulting in a PhD thesis in the physic-mathematical sciences by Viktor Ivanovič Šestakov, defended on September 28, 1938, at the State University Lomonosov, Moscow, Soviet Union [29]. In the thesis, Šestakov referred to the work on logic by Glivenko [9], and Žegalkin and Sludskaja [42]. The major part of the thesis of Šestakov was published in [30], [31] (Fig. 9). For discussions on the work by Šestakov, see [2], [3], [7], [11].

For historical accuracy, it should be noticed that the first remark on the applicability of the algebra of logic, whose central part is Boolean algebra, in logic network design, is due to the physicist Paul Ehrenfest as early as 1910. These remarks are presented in a review of the book Algebra of Logic by Louis Couturat (Table 1). The review was published in Žurnal Russkago Fiziko-hemičeskago Obščestva, Fizičeskij otdel (Journal of the Russian Physical-Chemistry Society), Part for Physics, Vol. 42, 1910, Second part, 382-387 [8].

The first report of this review by Ehrenfest in the western literature is ascribed to G.L. Kline [12] who pointed it out in 1951 in the review of a paper by S.A. Anovskaja [1].

Related remarks on the work of Ehrenfest were reported in an article by G.N. Povarov that was mentioned in 1959 in a review by Comey and Kline of a paper by Zinoviev [6]. Also, in a review by A. Church of a paper by T.A. Kalin [5], the same fact is pointed out with a statement that the author of the review had not seen the paper by Ehrenfest and had been informed about it by G.L. Kline.

In 1966, in [11], the following was stated

A 1910 book review by P. Ehrenfest [8] is sometimes mentioned as the first recognition that the algebra of logic might be used as an analytical tool
Figure 8: The page 10 of the paper by Nakashima in July 1941.
АЛГЕБРА ДВУХПОЛЮСНЫХ СХЕМ, ПОСТРОЕННЫХ
ИСКЛЮЧИТЕЛЬНО ИЗ ДВУХПОЛЮСНИКОВ (АЛГЕБРА А-СХЕМ):  

В настоящем работе установлено взаимное-однозначное соответствие между А-схемы, т.е. двухполюсными схемами, построенных исключительно на двухполюсниках, и А-выражениями, т.е. алгебраическими выражениями, определяющими соединения двухполюсников и связи друг с другом лишь двумя операциями: сложения и гармонического сложения.

Установленное взаимно-однозначное соответствие позволяет выстроить на основе А-выражения по заданному выражению А-схемы и, наоборот, позволяет сформулировать А-схему по заданному выражению А-выражения и производить последовательное преобразование А-схем (и частично, и упрощение А-схем) посредством алгебраических преобразований, соответствующих схемам А-выражений.

Дано понятие, что алгебра выраженных А-схем, т.е. таких А-сем, пропускающих которые работают от вольт-амперных схем, является алгеброй Бука. Отсюда следует, что для контрольных и формирования целей, которые могут быть нерациональными, такие алгебры Бука, может быть примениться весь аппарат алгебры Бука.

Двухполюсники $X_1$ и $X_2$ называются “соединениями друг с другом” или “от двухполюсников $X_1$ и $X_2$ построена схема”, или “двухполюсник $X_1$ и $X_2$ образуют схему”, если по крайней мере один полюс одного из них находится в электрическом контакте с полюсом другого из них.

Полюсы двухполюсников, между которыми существует электрический контакт, называются узлами схемы. Узел может быть, в свою очередь, полюсом, т.е. может допускать присоединение к нему других схем или источников электрической силы, но может быть и ток, что к узу дальше включить нельзя не допускается.

Графически узлы мы будем изображать посредством соединения полюсов, образующих узел, при чем узел, являющийся полюсом, будем изображать пустыми кружками. Узел же, не являющийся полюсом, будем обозначать черными кружками.

Для иллюстрации применены Фиг. 1, a и b. На Фиг. 1, a изображена такая же схема, что и на Фиг. 1, a, но переключается она уже не как двухполюсник, а как трехполюсник. В схеме Фиг. 1 допускается присоединение проводов не только к полюсам I и II, но также и к узлу III, являющемуся местом последовательного соединения двухполюсников $X_1$ и $X_2$. Не допускается соединения обеих полюсов одного и того же двухполюсника.

1 Настоящая работа представляет собой краткое изложение части кандидатской диссертации, защищенной автором 28 сентября 1941 г. в Московском институте Ленина государственным университетом им. М. В. Ленина.

Figure 9: The first page of the paper by V.I. Šestakov in Avtomatika i Telemekhanika, Vol. 2, No. 6, 1941, 15-24.
grants Ehrenfest this priority. Nakasima [17] is rarely mentioned in this
connection.

The work of Ehrenfest was also reported by H. Zamanek in 1993 in [38],
where it was stated

Paul Ehrenfest, the famous Austrian physicist and friend of Albert Ein-
stein, had postulated switching algebra as logical algebra already in 1910 - but
in Russian, in an unknown St. Petersburg journal of physics and chemistry,
and in a book review (of Couturat’s Logic). So his perfectly clear insight
remained unknown [8].

In [38] a reference to an earlier publication of Zemanek on the same
subject was given [39]. See also [36], [37], [40], [41].

References to the comments and translations of parts of the review of
Ehrenfest written by Zemanek in German are given in [13].

To the best of our knowledge, except this part of the review by Ehrenfest
that was translated into German by Heinz Zemanek [37], no translation into
English or other languages was published. With this motivation, in this
booklet, we reprint the review by Ehrenfest and provide a translation of
it into English accompanied by a brief analysis and discussion of related
references.

References

matematiki i matematiceskia logika (Foundations of mathematics and
mathematical logic)", Mathematika v SSSR za tridcat let 1917-1947
(Mathematics in the USSR for the thirty years 1917-1947), OGIZ,

brilliant idea authors”, Voprosy Istorii Estestvoznaniya i tehniki, No.
2, 2005, 112-121.

Context of University Philosophy, Canon+, Moscow, 2007.


[5] Church, A., "Review of Formal Logic and Switching Circuits, by


[16] Nakashima, A., ”Synthesis theory of relay networks”, The Journal of the Institute of Telephone and Telegraph Engineers of Japan, No. 150, September 1935, 731-752. Title translated also as ”The theory of relay


[38] Zemanek, H., "IFIP Universality - IFIP as a symbol and as a tool for information processing universality", in H. Zemanek (Ed.), *36 Years of IFIP*, A Volume Composed During the Presidency of Asbjörn Rolstadas, IFIP Secretariat, 1996, 43-53.


In [33], the following references on Ehrenfest and his remarks on the algebra of logic are provided
2 The Algebra of Logic by Louis Couturat

Table 1 shows the different editions of the book *The Algebra of Logic* by Louis Couturat (Fig. 10). The Russian edition of this book motivated Paul Ehrenfest to write a review of it, pointing out that the algebra of logic can be used as an underlying mathematical theory for the design of logic networks [8]. This review is reprinted and translated into English in Section 4.

The biography of Louis Couturat can be found in several publications. We refer to the probably most detailed among them

Table 1: Editions of the book by Louis Couturat.

<table>
<thead>
<tr>
<th>Edition</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2nd. edn., Paris 1914, 100 pages.</td>
</tr>
<tr>
<td>Hungarian translation <em>A logika algebraja</em>, translated by Denes Konig</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Price 90 kopejka (kopek)</td>
</tr>
<tr>
<td>English edition <em>The Algebra of Logic</em>, translated by Lydia G. Robinson</td>
<td></td>
</tr>
<tr>
<td></td>
<td>and Philip E. B. Jourdain, The Open Court Publishing Company,</td>
</tr>
<tr>
<td></td>
<td>Chicago, 1914, xiv + 98 pages, price $1.50</td>
</tr>
<tr>
<td></td>
<td>March 1915, 95-97.</td>
</tr>
</tbody>
</table>

30
Figure 11: The cover of the Russian edition of the book by Louis Couturat.
Алгебра логики.

1. Введение.

Основание алгебры логики положила Джордж Булл (George Bool, 1815—1864), развивал же и усовершенствовал ее Эрнст Шрёдер (Ernst Schröder 1841—1902). Основные законы этого исчисления были изобретены с целью дать выражение основных начал размышлений, законов мышления; но с чисто формальной точки зрения, которая свойственна математике, можно рассматривать это исчисление, как алгебру, основанную на некоторых произвольно установленных началах. Отличает ли это исчисление, — и, если отличается, то в какой мере, — действительными операциями мышления и может ли оно служить, так сказать, переводом размышлений или же зажигать его — это вопрос философский, который мы не будем здесь рассматривать. Формальное значение этого исчисления и интерес его для математика нисколько не зависит от интерпретаций, каким ему даются, и от приложений его к задачам логики. Мы будем, во всяком случае, излагать его как алгебру, а не как логику.

2. Две интерпретации логического исчисления.

Здесь представляется особенно интересное обстоятельство: эта алгебра допускает в самой логике и в различных, почти параллельных интерпретациях, в зависимости от того, выражают ли буны понятия, алгебра логики.
Figure 13: The title page of the French edition of the book by Louis Couturat.
Figure 14: The title page of the English edition of the book by Louis Couturat.
Figure 15: The title page of the Hungarian edition of the book by Louis Coutu-
rat.
3 Paul Ehrenfest

In the literature, the name of Paul Ehrenfest appears in different pronunciation as Ehrenfest, Erenfest, and Erénfést.

Paul Ehrenfest is a world renowned physicist whose main research interests were quantum theory, relativity theory, and statistical mechanics. For example, Ehrenfest is known for his work on the theory of phase transition of thermodynamic systems and for the Ehrenfest theorem in quantum mechanics. After publishing


Ehrenfest got a reputation for being among the first physicists to endorse the revolutionary theories of Albert Einstein with whom he later became a personal friend. Einstein appreciated Ehrenfest, particularly after he had
heard a lecture that Ehrenfest gave at the German University in Prague in 1910.

Most of the biographers of Paul Ehrenfest point out two facts that considerably influenced both his personal life and professional work. Paul Ehrenfest was an Austrian citizen of Jewish origin. This fact, combined with another fact of a similar kind resulted to a specific state of mind, leading finally to Erenfest’s tragic end by suicide. In order to marry an Orthodox Russian lady, Tatyana Alexeyevna Afanassjewa, a mathematician, both his wife and he had to declare themselves as nondenominational which was a way to avoid the rigid Austrian law regulations. At that time, such origins and religious backgrounds were not very helpful for finding a position at a university or good permanent employment as an engineer or scientist. The related difficulties and disappointments were the main characteristics of the first several years of the professional career of Paul Ehrenfest which can be summarized as follows.

Paul Ehrenfest received his Ph.D. degree at the University of Vienna in June 1904. Being jobless for two years, after unsuccessful attempts to find employment in Göttingen, Germany, where he and his wife were students, in the summer of 1907 they moved to St. Petersburg, Russia.

Thanks to his reputation as a well-known physicist, mainly due to his above mentioned paper from 1906, Ehrenfest established contacts with physicists in St. Petersburg and with the very famous mathematician Vladimir Andreevich Steklov. This, however, did not help him to get a permanent position at the University of St. Petersburg in spite of his efforts to establish the necessary links. The major obstacles were the above mentioned facts of his origins and personal life that were strongly opposed by Russian society at the time, as well as his criticism of the old-fashioned way of work at the university and the rigid study procedure.

Due to an invitation by Steklov, Ehrenfest gave several lectures at the University of St. Petersburg on different mathematical subjects. Ehrenfest also became a member of the editorial board of the Journal of the Russian Physical-Chemical Society, being especially engaged in publishing a supplement of this journal entitled Problems in Physics. While serving as a member of the editorial board, Ehrenfest regularly attended meetings of the Russian Physical-Chemical Society and published several articles and book reviews including the review translated and reprinted in this booklet.

In 1909, Ehrenfest worked at the Polytechnic Institute for almost a year, teaching differential equations of mathematical physics for two semesters. Disputes about his way of work and the procedures at the university com-
bined with his personal background and related prejudices present in Russian society at the time resulted in his dismissal from the Polytechnic Institute.

Over a period of several years, Ehrenfest tried to find a position at different universities including the University of Czernowitz (now Tsjer-

novtsi), Ukraine, in 1910, and three universities in Germany, the University of Leipzig, the University of Munich, and the University of Berlin. Although highly recommended and supported by many important physicists, his at-

tempts remained unrewarded.

It should be noticed that after attending a meeting of Zionists in Vienna in 1910, Ehrenfest became interested in this movement and highly enthusi-

astic about it.

Ehrenfest left St. Petersburg on January 6, 1912, traveling to Berlin to meet Max Planck and discuss two of his important papers with him


During the same trip, Ehrenfest visited several famous physicists while traveling to Munich, Zurich, and Prague, where he met Einstein.

When he returned to St. Petersburg in early March 1912, Ehrenfest found an opportunity to get a position as the Chair of Theoretical Physics at the University of Leiden, as the successor of Hendrik Antoon Lorentz. Ehrenfest worked there until his tragic end. For more details on the biogra-

phy of Paul Ehrenfest, we refer the reader to


Hollestelle, M.J., "Paul Ehrenfest as a mediator", in M. Kokowski, (Ed.), The Global and the Local: The History of Science and the Cultural Integration of Europe, Proceedings of the 2nd ICESHS, Cracow, Poland, September 6-9, 2006, 787-792.

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Figure 17: Paul Ehrenfest while teaching (photo taken from Mac Tutor archive).
When presenting formal logic, it is necessary to know the following circumstances. The extraordinary tight classification of different types of reasoning and syllogisms, that is already developed in knowledge, founds in language a very difficult and inaccurate instrument to express it.

For this reason, in the theory of reasoning and syllogisms, it was accepted long ago that this classification should be expressed by conditional symbols. This primarily concerns the subclassification of syllogisms into type "A, E, I, O" and the derivatives by reasonable symbols for 19 forms of regular syllogisms (from "Barbara" to "Ferison" in the 13th century). Later, a symbolic to represent different notions by circles in a plane was developed. The different ways of the mutual placement of circles correspond to different cases of combining two notions (premises) into a single conclusion.

The first symbolic notation, hardly better than stenography, unifies in a common picture all of the members of the syllogism, however, it is inflexible. The second symbolic notation is already considerably better: over a system of such circles, it is possible, after defining the corresponding rules, to perform transformations that also have a defined interpretation in logic. (This can be compared with the fact that in chemistry formulas permit not only a systematic registration of different substances, but besides that, the transformations of formulas by predefined rules correspond to chemical reactions.)

It is easy to understand that this area will sooner or later lead a speculative-mathematical mind to the following question: Can these principles of symbolic notation, that appeared so fruitful in operations over numbers and quantities - "symbolic notation" - be transferred into operations over all concepts?

Now, regarding the attempts by Leibniz and Grassman. A wider development of the "Algebra of Logic" is due to two mathematicians: the Englishman George Boole (1815-1864) and the German E.
Schröder (1841-1902), to whose further development mathematicians from all over the world contributed. In particular, the Russian mathematician P. Poreckij greatly assisted in the simplification of methods by means of his original formulation of the problem.

The "Algebra of Logic" first of all establishes a symbolic notation for these elementary actions which appear to be essential in operating with notions, similar to addition, equivalence, etc., which are used in operating with numbers and quantities. Further, the axioms upon which the entire formal logic is based, converts into the form of rules how to perform computations over these symbols, i.e., how a multitude of such symbols can be transformed into another multitude equivalent to it.

From understanding, if it is possible to say in this way, of the typographic character of these operations, we select exactly those symbols that were already long ago - with a completely different meaning - introduced into printing by mathematicians. This compliance - without which the review of the book on the algebra of logic on the pages of this journal would be impossible - often gives to the formulas in this discipline a paradoxical form at first sight, e.g.,

\[
\begin{align*}
1 + 1 &= 1, \\
A + AB &= A, \\
AAA &= A, \\
(A + \Gamma)(\Gamma + B)(B + A) &= A\Gamma + \Gamma B + BA,
\end{align*}
\]

where the identities (2), (3), and (4) hold for an arbitrary choice of notions \(A\), \(\Gamma\), and \(B\). For example, \(A = \) all that is black, \(\Gamma = \) all that is colored, \(B = \) all that is firm.

The correctness of these equalities becomes understandable at the very same moment when the meaning of the operations used in the algebra of logic behind these symbols is explained.

\((A\Gamma)\) denotes "all who belong at the same time to the class \(A\) and the class \(\Gamma\) (black colour)."

The same applies to \((\Gamma B)\) and \((BA)\).

But, it is slightly difficult to use languages in these - from the point of view of logical relations - primitive constructions. In most cases, languages prohibit some propositions (coloured blackness?!) In other cases, the proposition - under the influence of different, arbitrarily added agreement - gives a completely different meaning to the words (shine silk = silk’s shine).
(A + \Gamma) denotes all who belong to class A and also all who belong to class \Gamma.

The most correct and reasonable expression of this "addition" would be, please, the following: all that belong to either A or B, or both at the same time (for example, "physicians and scholars").

It is now easy to verify this on examples of equalities (2) and (3).

For example, all coloured + all black balls = all coloured.

With the help of "multiplication" and multiple application of equalities (2) and (3), it is easy to verify (4).

1 denotes: the universe of all thinkable thoughts.

After that, equality (1) is obvious.

0 denotes: classes, that do not contain any thinkable thoughts. A' denotes "not A", i.e., all that are not A.

It is easy to verify the following statements:

1. \(AA' = 0, A + A' = 1\),

2. \((A\Gamma)' = A' + \Gamma'\)

\((A < \Gamma)\) denotes: all A smaller than \(\Gamma\), which, however, can be expressed as:

\[A = x\Gamma \text{ or } A\Gamma' = 0.\]

By using such expressions it is possible to express all the syllogisms in an entirely numerical manner. For example, the form of the syllogism "Ferison"

\[\text{No man (L) is clairvoyant (M).} \quad L = xM'\]
\[\text{Some people are scholars (H).} \quad yL = zH\]
\[\text{Some scholars are not clairvoyant.} \quad zH = xyM'.\]

By using the algebra of logic, it is possible to treat all syllogisms without intermediate constructions. In general, it is possible to reach the goal faster: first in the form of equalities we establish the entire system of given parcels. This system of reasoning is transformed into a unique system i.e., into an equivalent system, paying attention that the equality \(A + \Gamma + B + \cdots = 0\) is equivalent to \(A = 0, \Gamma = 0, \ldots\).

Furthermore, by using predefined rules, we calculate a system of reasoning - in some sense complete (!) - that follows from this given central reasoning. All of the computations are very simple, since in the algebra of
logic - unlike classical algebra - action spaces do not spread until infinity. (For instance, "exponentiation" does not exist here - see equality (3)).

The area of application of the algebra of logic is further considerably extended due to the following observation.

In equalities in the algebra of logic, symbols $A, \Gamma, \ldots$, can represent particular notions, but also entire equalities that connect notions $L, M, \ldots$.

In this interaction, the equality

$$(LM' = L) = (LM = 0)$$

expresses in an unexpectedly short form the following theorem that can easily be verified on particular examples:

"A statement that the set of all $L$ that are at the same time not $M$, is identical to the whole set $L$, is equivalent to the statement that none $M$ belongs to the set $L".$

In the same way, the equality

$$(A + \Gamma + B = 0) = (A = 0)(\Gamma = 0)(B = 0)$$

formulates the above theorem on joining several logic equalities into a single equality.

The goal of all these remarks is to give a description from another angle of a discipline which was introduced in the not very extensive book by Couturat.

It can be considered as an introduction to the algebra of logic in the sense that the author does not assume any background knowledge, except familiarity with general notions of logic, and at the same time presents to the reader all the questions that are foundations of a very extensive (the work of Schröder consists of four large volumes) literature on this subject.

Reading this book, on the other hand, requires very serious work, since the author does not restrict the presentation to the general presentation of the symbolic method, but uses it - already from page 7 - for the presentation and derivation of all the theorems. In this manner, this book cannot just be read, it is required to perform computations over entire pages, while reading. Besides that, the point of view presented in the book is at a high level of abstraction, the book is written for a French reader - a mathematician. (In this latter respect, additional remarks in the Russian edition make it considerably easier for the Russian reader.)

For an initial familiarization with the subject, it could be preferable to read several former publications. The presentation of the symbolic method

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3 For example, E. Schröder, *Operationskreis des Logikcalculus*, Teubner, 1877.
in this book, on the other hand, offers the reader a possibility to be convinced that this method in any case has three advantages:

1. The book offers a possibility to distinctively denote the set of all propositions, upon which some conclusions are based, in such a way that the introduction of unconscious assumptions, that are often met in the formulation of reasoning, are almost completely excluded. In reality, axioms upon which formal logic is based appear in distinct and completely unusual forms.

2. Consequently, the formulation of any reasoning in terms of logic equalities is at least 5 to 10 times shorter than the literal formulation, which is an admirably concise presentation.

3. The symbolic formulation provides the possibility of ”computing” conclusions from such complex systems of propositions, for which a literal presentation is almost or completely impossible.

Fortunately, we have already lost the habit of requiring that each mathematical speculation needs ”practical usefulness”. It is however not less appropriate to tackle the question whether in physics or technique such complex systems of propositions exists. We think that we should answer these questions affirmatively. Example: Let the task be to design networks of connections in automatic telephone stations. It is necessary to determine the following: 1) If the station will work correctly for an arbitrary combination of possible occurrences in the working station. 2) If the station contains some redundancies.

Each of these combinations is a proposition, each small commutator is a logic ”Or-Or”, all together - a system of qualitative (non quantitative) ”propositions”, leaving nothing more to be desired regarding complexity and intricacy.

Does it follow that, when solving such problems, every time some ingenious method - in many cases just a simple routine method - of trials on a graph should be used?

Is it right, that regardless of the existence of the already elaborated algebra of logic, the specific algebra of switching networks should be considered as a utopia?

P. Ehrenfest
5 A Reprint of the Review by Ehrenfest

This section contains the reprint of the original review of P. Ehrenfest as it appeared in


Библиография.

А. Кутюра. Алгебра Логики. Перевод съ французскаго съ прибавлениемъ проф. И. Семеновскаго. Mathesis. 1909. 104 ± XIX стр. (цена 90 к.).

При изложенні формальной логики даєть себѣ знать слѣдующее обстоятельство: чрезвычайно тонкія классификація раз-

1) Упомянутый впервые былъ демонстрированъ у насъ А. Ф. Гершуномъ въ заключеніе Р. Ф. О., 12 октября 1910 г. Въ настоящее время его уже можно получить въ Петербургѣ—въ шведскомъ промышленномъ магазинѣ, Литейный, 24.

This page presents page 382 of the review, the text on pages 383 to 387 is continued on the following pages. 4

личных типов суждений и умозаключений, выработанных уже в создании, находить в языке описком тщательный и точный инструмент для своего выражения.

Потому в учении о суждениях и умозаключениях уже давно привык передавать эту классификацию условными символами.

Сюда относятся прежде всего подразделение суждений на типы \( A, E, I, O \) и произведенные из них слоговые символы для 19 форм правильных умозаключений (от \( \text{"Барбара"} \) до \( \text{"Фермин"} \) с 13-го столетия). Подобное развитие символики, представляющая различные понятия при помощи кругов на плоскости, различные способы относительного расположения кругов соответствуют различным случаям соединения двух понятий в одно суждение.

Первая символика едва ли больше, чем естественная, объясняяющую в одну картину все члены суждений, но неясная. Вторая символика дает уже гораздо больше: дать системой таких кругов можно по определенным правилам производить преобразования, имеющие также определенную интерпретацию в логике. (Сравнить с этим тот факт, что формулу \( \text{"ини"} \) доставляют не только систематическую регистрацию различных вещей, но что кроме этого—преобразования этих формул, произведенные по определенным правилам, соответствуют логическому преобразованию).

Само собой разумеется, что эта область ранее или позднее должна была побудить философскую математику умы к выделяющей понятия, т. е. принципу символических обозначений, который оказался столь плодотворным для действий над числами и величинами,—"буквенное исчисление"; верности над действиями над всеми понятиями.

Сюда относятся уже понятия Лейбница и Гессена. Более широко развитием обязано \( \text{"алгебра логик"} \) двум математикам: интересуясь Джорджем Буллом (1815—1884) и нынешним Е. Шредером (1841—1902); в дальнейшей разработке принимали участие математики всех стран. В частности русский математик П. Порфирий много создавал методы и введение в эту область оригинальной постановкой вопроса.

"Алгебра логик" прежде всего устанавливает символические обозначения для т. е. элементарных действий, которые являются такими же основными при оперировании с понятиями, как и сло-
женіе, приравниваний і т. і. при оперировахів съ числами и величины.

Далее, акценты на которых основаны все формальна логика, обозначены въ форму правилъ о томъ, какъ надъ этими символами проводить вычисления, т. е. какъ одинъ комплексъ такихъ символовъ преобразовать въ другой, эквивалентный ему.

Изъ соображений, если можно такъ выразиться, типографского характера для этихъ операций выбраны тѣ самыя знаки, которые уже давно—съ совершенно инымъ значениемъ—были математиками введены въ печать. Это соглашение,—безъ котораго на страницахъ настоящаго журнала была бы невозможна рефератъ по алгебры логики,—придется формуламъ этой дисциплины неоднократно на первый взглядъ парадоксальнымъ видъ, напр.: (1) $1 + 1 = 1$; (2) $A + A = A$.

(3) $A A A = A$; (4) $(A + B) (B + A) (B + A) = A B + B A + B A$.

Примѣромъ равенства (2), (3) и (4) итого можно при произвольномъ выборѣ понятий $A$, $B$, и $C$. Напр., $A = «все, что черно»$, $B = «все, что шарообразно»$, $C = «все, что твердо»$.

Но справедливость этихъ равенствъ точна настолько, какъ только будешь объяснено, какъ операции подразумеваются въ алгебре логики подъ значками, обозначающими дѣйствія.

$(A B)$ обозначаетъ: «все, что одновременно принадлежитъ классу $A$ и классу $B$ (черный шаръ).»

Тоже относительно $(B A)$ и $(A B)$.

Впрочемъ, языкъ слишкомъ тяжеловѣснъ, чтобы доступить за этой—въ логическомъ отношеніи—примитивной конструкціи; въ большинствѣ случаевъ она запрещаетъ перестановку (шарообразный черный шаръ?); въ другихъ случаяхъ перестановка—подъ влияніемъ различныхъ, случайно сложившихся обстоятельствъ, придаетъ словамъ совсѣмъ другой смыслъ (ослѣдствіе желъ = шелковый блескъ).

$(A + B)$ обозначаетъ: «все, что принадлежитъ классу $A$, и сверхъ того все, что принадлежитъ классу $B$.»

Найболѣе точное словесное выраженіе этого «сложенія» будетъ, пожалуй, слѣдующее: все, что принадлежитъ или $A$, или $B$, или обоимъ вмѣстѣ (примѣръ: «врачи и ученые»).

Теперь нетрудно на примѣрахъ провѣрить равенства (2) и (3). Напр., все шарообразное $+$ всѣ черные шары $=$ всѣ шарообразные.

При помощи „умноженія“ и многократнаго прикладенія равенства (2) и (3) легко провѣрить (4).
1 обозначает: совокупность всего мыслимого.
Поскольку это равенство (1) очевидно.
Оно обозначает: класс, не содержащий ничего мыслимого.
$A'$ обозначает «Не—А», т. е. все, что не есть $A$.
Легко проверить следующее утверждение:
$$AA' = 0; \quad A + A' = 1.$$

$$(AB)' = A' + B'.$$

$(A < B)$ обозначает: вся $A$ суть $B$, что, впрочем, можно выразить и так:
$$A = xB \text{ или } AB = 0.$$  

С помощью таких приемов можно уже чисто вычислительным путем вывести все аналогии, напр., форму «ферзь».
На одном конце (L) не есть $xM'$.  
Некоторые люди—ученые (H) . . . 
Некоторые ученые не люди . . . 
$$L = xM', \quad L = xH, \quad zH = zH.$$

Но, пользуясь алгеброй логики, можно обойтись без последовательного построения всех аналогий. Обыкновенно можно коротко коротко прийти к цели: сперва в форме равенства устанавливаются все системы сутузений. Эта система суждений преобразуется в одно единственное, имеющее эквивалентное; при этом принимается во внимание, что утверждение $A + B + B + \ldots = 0$ равносильно с $A = 0, B = 0, \ldots$

Далее по определенным правилам вычисляется в известном смысле—полная (l) система суждений, которые следуют из этого центрального суждения. Все вычисления очень просты, так как в алгебре логики—в противоположность обыкновенной алгебре—круг действий не разрастается до бесконечности. (Напр., здесь не существует «отрицания» см. равенство (3)).

Область применимости алгебры логики значительно расширяется еще в виду следующего соображения.

В равенстве $\lambda$ алгебры логики символы $A, B, \ldots$ могут представлять не только отдельные понятия, но и целые равенства, связанные понятия $L, M, \ldots$

При такой интерпретации равенство
$$(LM' = L) = (LM = 0)$$
выражает в неожиданно короткой форме следующую теорему, которую легко проверить на частных примерах:  

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«Утверждение, что совокупность всех \( J \), которая в то же время не—\( M \), совпадает с совокупностью всех вообще \( J \), равноценно с утверждением, что ни одно \( M \) не принадлежит к классу \( J \).

Точно также равенство:

\[ (A + B + C = 0) = (A = 0) \quad (B = 0) \quad (C = 0) \]

формулирует вышеназванную теорему о соединении некоторых логических равенств в одно.

Все эти указания имеют целью хотя бы с вида ших стороны охарактеризовать технику своеобразной дисциплины, введением в которую является небольшая книга Кутюра.

Введение в алгебру логики она может считаться в том смысле, что автор не предполагает у читателя никаких предварительных знаний, за исключением знакомства с общими понятиями логики, и в то же время показывает читателю все вопросы, которые легли в основу весьма обширной (исчисление Шредера обнимает 4 больших тома) литературы этого предмета.

Но с другой стороны чтение этой книги требует весьма серьезной работы, так как автор не ограничивается общим изложением символического метода, а пользуется ими—написан в 7-ой страницы—для изложения выводов всех теорем. Так что образец этой книги нельзя просто прочитать, а необходимо на протяжении целой страницы сопровождать чтение ее производством вычислений. Кроме того, точка зрения ее чрезвычайно абстрактна—она написана для французского читателя—математика. (В последнем отношении примечания русского издания представляют весьма ценное облегчение для читателя).

Поэтому для общего ознакомления с предметом придается, может быть, предпочтение некоторым прежним сочинениям 1). Но с другой стороны как раз это изложение по символическому методу дает читателю возможность убедиться, что он во всяком случае имел три преимущества:

1. Он дает возможность отчетливо обозначить всю совокупность предложений, на которых основываются какиелибо выводы, так что введение безосновательных допущений, так что часто встречающееся при словесной формулировке рассуждения, почти совершенно исключается. В частности, аксиомы, на которых

1) Напр., E. Schröder, Operationskreis des Logikrechns. Teubner. 1877.
основывается формальная логика, выступает в отчетливо и
притом совершенно непривычной форме.
2. Вследствіе того, что формулировка великой разсужденія
при помощи логическихъ равенства по крайней мѣрѣ въ 5—
10 разъ короче словесной, достигается удивительная сжатость
назначенія.
3. Символическая формулировка даетъ возможность «вычислять»
слѣдствія изъ такихъ сложныхъ системъ посылокъ, въ которыхъ
при словесномъ изложеніи почти нѣть возможности разобраться.
Къ счастью, уже отвѣчию требовать отъ каждой математиче-
ской спеціализации прежде всего «практической пользы». Тѣмъ не
менѣе, быть можетъ, умѣстно коснуться вопроса о томъ, не встрѣтъ-
чаются ли въ физикѣ или въ технікѣ въ самомъ дѣлѣ такихъ слож-
ныхъ системъ посылокъ. Мнѣ думается, что на этотъ вопросъ
следуетъ отвѣтить утвердительно. Примеръ: пусть имеется
проектъ схемы проводовъ автоматической телефонной станціи.
Нужно опредѣлить: 1) будетъ ли она правильно функционировать
при любой комбінаціи, могущей встрѣтиться въ ходѣ дѣятель-
ности станціи; 2) не содержать ли она наивникъ узловъ.
Каждая такая комбінація является посылкой, каждый малень-
кий коммутаторъ есть логическое «или» или воплощенное въ
абонентѣ и латунѣ, все вместе—система чисто качественныхъ (въ
стѣны слабаго тока именно чею качественныхъ) «посылокъ», ни-
чего не оставляющая желать въ отношеніи сложности и за-
датности.
Сдѣланъ ли при рѣшеніи этихъ вопросовъ разъ навсегда
уколеблівиться гениальнымъ—а по большей части просто рути-
ными—способомъ пробованія на графикахъ?
Правда ли, что посмотря на существованіе уже разработанныхъ
«алгебры логики» своего рода «алгебра распределительнѣйъ
схемъ» должна считаться уловкой?

Н. Эренфестъ.

А. Б. Цыганковъ. Начальная физика. Первая ступень. XX+489.
стр. Москва.
"Первая ступень" курса физики, разработанного авторомъ на
двѣ части, представляетъ собою попытку дать элементарный
очеркъ физики, имеющий целью вмѣстѣ съ изложеніемъ немногого
6 Reviews about Ehrenfest

In this section, we reprint reviews about the work of scholars where the remarks of P. Ehrenfest on applicability of algebra of logic in the design of logic networks is mentioned. These are reviews of

1. Alonso Church about the article by of T.A. Kalin,
2. Alonso Church about papers by A. Nakashima and M. Hanzawa,
3. D.D. Comey and G.L. Kline about the article of A.A. Zinoviev,
4. G.L. Kline about the article of S.N. Anovskaa.
6.1 Review by Alonso Church for T.A. Kalin mentioning Ehrenfest


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"The historical development of the application of symbolic logic to calculating machinery is traced to the present, and a brief sketch of some modern developments is presented. " The author recounts briefly, in descriptive fashion, the invention by Charles Babbage of a scheme of mechanical notation by which the various parts of a mechanism are expressed on paper by symbols, and the development of the algebra of logic by Boole and Jevons (only these two names are mentioned in the latter connection). Shannon's IV 103(3) (reprinted XVIII 347(1)) is referred to for the application of Boolean algebra or propositional calculus — the distinction between them is not important in this connection — to relay and switching circuits, and Hartree's *Calculating instruments and machines* (XVIII 347) is mentioned as using a like method (in connection with computing and control circuits). The final half of the paper under review is then devoted to an account in more detail of the content of *Synthesis of electronic computing and control circuits* (XVIII 347), which uses, not Boolean algebra in the modern sense, but a numerical representation of it which is in fact the same as the original algebra of Boole (191, 2, 3). The author concludes: "There can be little doubt that formal symbolic design methods will grow in utility and neatness of application as new circuitual techniques are adopted and as new switching elements are discovered and put to work. It is almost true by definition that digital systems operating by means of 'on-off' devices will continue to offer a most enticing challenge to two-valued logic, and it is our task to so refine our exact methods that the designer shall be increasingly free to devote his skill and experience to areas less subject to routine effort."

The author's statement that Shannon's paper of 1938 is the first exposition of the relations between "two-valued logic and switching circuitry" requires some qualification. It seems to be not generally known that the first suggestion of such a relation-
ship was made in 1910 by Eréndéfö, in his review (1864) of Couturat's 10020A. According to Anovcač (XVI 46), details of Eréndéfö's proposal were worked out by Šestakov in 1934–35 but not published until 1941. Meanwhile the same idea had been reached independently by Nakasima and Hanzawa in 1936 (see the review next following).

The reviewer has not seen Eréndéfö's review, and is indebted to George L. Kline for information as to its content. He is also indebted to George W. Patterson for calling his attention to the papers of Nakasima and Hanzawa, and others published in Japan.

Alonzo Church


*Nippon electrical communication engineering* publishes condensed English translations, and abstracts in English, of papers which were previously published in Japanese in the *Journal of the Institute of Electrical Communication Engineers of Japan*. The first of the above papers, for example, is described as a condensed translation of a paper which appeared in two parts in the latter periodical, no. 165 (December 1936) and no. 167 (February 1937). The reviewer has not seen the Japanese originals of the papers.

The six papers are concerned with developing and applying an algebra of partial paths in relay circuits, which is in fact identical with the "symbolic relay analysis" that was later introduced by Shannon, and dual to the "algebra of switching circuits" of Eréndéfö and Šestakov (see the preceding review).

The first paper introduces the algebra by providing that if \( A \) and \( B \) are simple partial paths (two-terminal circuits), then \( A + B \) shall represent the series connection of \( A \) and \( B \), and \( AB \) the parallel connection of \( A \) and \( B \); \( A = B \) shall mean that the acting functions of \( A \) and \( B \) are equal, i.e., that \( A \) is open when \( B \) is open and closed when \( B \) is closed; \( A \) shall be a simple partial path which is open when \( A \) is closed and closed when \( A \) is open; \( p \) and \( s \) shall be simple partial paths which are always open and always closed respectively (or, as the authors say, give always infinite impedance and zero impedance respectively). Many laws of the algebra are developed which in fact coincide with familiar laws of Boolean algebra, but the authors do not state that the algebra is a Boolean algebra.

In the third paper (of which the Japanese version was published in August 1937) the algebra is reduced to an algebra of sets by making correspond to each simple partial path the set of (in effect) times at which its impedance is infinite, so that "theorems and expressions developed in the theory of set may, therefore, be applied to acting impedance problems of simple partial paths." In the sixth paper the authors make explicit reference for the first time to Boole (194) and Schröder (427); the expansion theorem mentioned in the title of this paper is Boole's law of development (194, pp. 72–75), as the authors point out.

Alonzo Church
6.2 Review by Alonso Church for A. Nakashima and M. Hanzawa


Akira Nakashima. Algebraic expressions relative to simple partial paths in the relay circuit. Ibid., no. 12 (September 1938), pp. 310–314.


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The six papers are concerned with developing and applying an algebra of partial paths in relay circuits, which is in fact identical with the "symbolic relay analysis" that was later introduced by Shannon, and dual to the "algebra of switching circuits" of Erméfjést and Sztakow (see the preceding review).

The first paper introduces the algebra by providing that if A and B are simple partial paths (two-terminal circuits), then \( A + B \) shall represent the series connection of A and B, and \( AB \) the parallel connection of A and B; \( A = B \) shall mean that the acting functions of A and B are equal, i.e., that A is open when B is open and closed when B is closed; \( A \) shall be a simple partial path which is open when \( A \) is closed and closed when \( A \) is open; \( \phi \) and s shall be simple partial paths which are always open and always closed respectively (or, as the authors say, give always infinite impedance and zero impedance respectively). Many laws of the algebra are developed which in fact coincide with familiar laws of Boolean algebra, but the authors do not state that the algebra is a Boolean algebra.

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6.3 Review by D.D. Comey and G.L. Kline for A.A. Zinoviev mentioning Ehrenfest


A. A. Zinov'ev. Rasširat' tematiku logicheskih issledovanij (Broaden the subject matter of logical investigations). Ibid., no. 3 (1957), pp. 211–215.


Zinov'ev's two papers, although published nearly a year apart, constitute a single report on the logic seminar of the Institute of Philosophy in Moscow. This seminar was organized in September 1956 (i.e., at the high point of the Soviet cultural "thaw" of that period). Zinov'ev explains that such a seminar was needed because, "until very recently there has been little attention to the development of logic in philosophical institutions; the range of interests of professional logicians has been narrowed to the limits of the content of obsolete textbooks" [i.e., Soviet texts in purely Aristotelian logic] (p. 211).

Since a chief purpose of the seminar was to bring Soviet philosophers up to date, many of the reports are historical in character. Ruzavin and Stäfkin survey the development of mathematical logic from its beginnings in Stoic and medieval logic. Finn and Lahiri, in a joint paper, outline recent British and American work in the theory of induction, modal logic, the logic of explanation, etc. Later reports are of greater theoretical interest: Pobarov discusses combinatorial logic, distinguishing "technical logic" from the "algebra of logic," and pointing out that "the possibility of technical applications of mathematical logic" (i.e., the application of Boolean
algebra to the analysis of electric relay-contact circuits) was first noted by the Russian physicist P. S. Erőfű in 1910 (cf. the review XVI 46). Stáškin, in a second paper, explores the status of logical and semantic paradoxes. He leaves open the question as to whether they can be resolved by the elimination of "semantic terms" (p. 163). But he explicitly denies that all semantic paradoxes can be resolved merely by distinguishing between "use" and "mention" of expressions, or by modifying the rules for logical negation.

Lahuti relates Ackermann's "strengte Implikation" (cf. XXII 327) to Russell's "material implication" and Lewis's "strict implication," asserting that Ackermann's, and Čerstvčik's, method make it possible to eliminate most, though not all, of the paradoxes in SI. Ackermann's system has the further merit of accommodating supplementary non-logical axioms and of being extendable to the predicate calculus. — Povarov discusses group invariance, including symmetry, of Boolean functions. — Finn reports on information theory and "machine logic," distinguishing a general information language from special information languages — those of geometry, chemistry, etc. He symbolizes the "logical processing of information" as follows: $Q_1 \ldots , Q_n \vdash i; Q_1 \neq i, Q_2 = i, Q_{n+1} \neq i$, where "$Q$" is the "coded" message and "$i$" is the desired information. Finn also distinguishes several kinds of "functors," e.g., "operators" (which form names from names), "predicators" (which form propositions from names), "connectors" (which form propositions from propositions).

Zinov'ev's own paper on "paradoxes of indefiniteness" is reproduced in fullest detail. It sets forth a system to avoid the paradox of the set of all normal sets and other related antinomies of set theory. However, Zinov'ev's system uses such a vague criterion of existence for sets (given in terms of an equally obscure concept of the existence of a proposition), that the paper itself suffers from indefiniteness. For instance, the definition of a set on page 171 presupposes, in the definiens, the concept represented by the definiendum. Again, on page 170 it is stated that if $II$ does not exist, then every statement containing $II$ is indefinite; but a few paragraphs later we are told that if $II$ does not exist, then both the statements $II$ does not exist and $II$ exists are definite, and are the only two definite statements containing $II$. Yet it is plain that any number of disjunctive propositions containing $II$ does not exist as one member would be definite by Zinov'ev's original criterion.

Zinov'ev often fails to distinguish between use and mention of an expression, and, although he prescribes "non-effective" proofs by reductio ad absurdum, he appeals to the principle that either $X^i$ does exist or it does not. The proof given on pages 171–172 is invalid: The author constructs a sequence of sets $M_1, M_2, \ldots , M_{i-1}$ such that for each $i$ there is a property $X$ which holds for $M_1, M_2, \ldots , M_{i-1}$, but does not hold for $M_i$. From this he erroneously infers that there cannot be any property $X$ which holds for all the sets $M_1, M_2, \ldots , M_1, M_2, \ldots$. In fact, if $M^2$ is taken as the null set and $M^3$ as the set containing only one member, then both $M^2$ and $M^3$ are normal. Using only these two sets, we can go on to form $M_1, M_2, \ldots$ according to his method. If we then take our construction as the construction of the integers in set theory, $M^1$ is zero, $M^2$ is unity, and $M_1, M_2, \ldots$ are two, three, \ldots. Quite obviously there are properties which hold for all of $M^1, M^2, M_1, M_2, \ldots$; e.g., they are all integers; they are all less than two or else uniquely factorable into prime factors.

These Soviet discussions reveal a detailed familiarity with current work in mathematical logic and information theory outside the Soviet Union. They are also marked by a striking preference for non-Russian technical terms (e.g., antécédente, description, distinction, informatic, interprétacié, kod [code]) in cases where there are perfectly good Russian equivalents.

David D. Comey and George L. Kline
6.4 Review of G.L. Kline for S.N. Anovskaja


The present paper is a survey of the work in mathematical logic and the foundations of mathematics done in the Soviet Union between 1917 and 1947, with some reference to pre-revolutionary mathematicians and logicians. Soviet mathematicians, the author emphasizes, reject the view of Poincaré, Heyting, et al., that “the propositions of pure mathematics say nothing about reality.” On the Soviet view, the formal axiomatic systems of mathematics—which admit of many qualitatively different interpretations—rest on a material basis (назади реал — наказлтных) arithmetic, in which numbers and the relations between them are univocally defined. The spatial forms and quantitative relations of the material world are the specific subject-matter of mathematics (Engels). In contrast to Carnap and the logical positivists, A. N. Kolmogorov asserts that a formal apparatus is valid only when it “corresponds to a real content.”

Turning to the historical development of mathematical logic, the author mentions the work of De Morgan, Boole, Jevons, Peirce, and Schröder, and goes on to state that “the culmination of this period... was the work of the Russian logician, P. S. Porččkij, Lobačevskij’s colleague at Kazan University” (p. 19). Porččkij considered his 1926 the first attempt at a complete theory of qualitative inference (by a “quality” Porččkij meant what is now called a “one-place predicate”).

Soviet mathematicians, according to the author, deny that there is any “crisis” in the foundations of mathematics, although they recognize real difficulties in connection with the applicability of the laws of formal logic, extrapolated from finite domains, to infinite domains, especially the law of excluded middle; and the paradoxes of mathematical logic and set theory. The chief interest of Soviet writers, we are told, is in the application of mathematical logic to special problems in mathematics and technology.

One of the first treatments of the law of excluded middle, according to Ānovskaja, is to be found in the introduction to S. O. Šatunovskij’s algebra textbook, published in Odessa in 1917. From the excerpts which she reproduces, it is evident that Šatunovskij did have ideas on the subject similar to those of Brouwer, although they seem to be less clearly formulated. However, it should be noted that Brouwer’s publications along this line go back to 1908.
The problem of the excluded middle was discussed in 1925 by Kolmogorov in his 314t, which, the author maintains, anticipated Gödel's result of 1935. To the effect that "die intuitionistische Arithmetik und Zahnentheorie nur scheinh organs as die klassische." Kolmogorov offers an interpretation of classical arithmetic for which—through replacement of every variable by its double negation—all of its "known" propositions become propositions of intuitionistic arithmetic. He further states: "The application of the principle of excluded middle will never lead to a contradiction. In fact, if a false formula were obtained with its help, the corresponding formula of pseudo-mathematics would be demonstrated without its help, and would nevertheless lead to a contradiction." (314t, p. 661.)

In 1929 V. I. Glivenko in his 3312 obtained a stronger result than Kolmogorov's regarding the propositional calculus. He showed that if a formula is demonstrable in the classical propositional calculus its double negation is demonstrable in the intuitionistic calculus, and that if the negation of such a formula is classically demonstrable it is also intuitionistically demonstrable.

Glivenko emphasizes that the intuitionistic logic, formulated as a finite set of axioms and rules of inference, can be "adequately applied to a given domain of objects" quite independently of the epistemological premises of intuitionism. Soviet writers accept the calculus and build upon it, but they reject the "idealistic" epistemology. The real significance of Heyting's logic—as a calculus of problem-solving—was exhibited by Kolmogorov in his 341t.

The work 304t of Ščuvnikov's pupil, M. I. Schönfinkel, written in 1929 and published in 1934, is justly stressed as a major advance in the development of mathematical logic. However, Anokhin also credits Schönfinkel with originating the idea of a function as a special abstract object distinct from its values; in fact, this notion goes back at least to Frege (though adapted with some in the terminology which is somewhat misleading). The author states that Schönfinkel's ideas were widely taken up by American mathematicians, first by Curry, who constructed his 'combinatory logic' (1930) on their foundations, and by Church, whose calculus of λ-conversion represents a certain 'formalization' of Schönfinkel's ideas" (p. 33). Without minimizing the significance of Schönfinkel's radically new idea, it should be noted that Curry has carried its development to a point far beyond the bare beginning made by Schönfinkel, and has contributed important additional ideas without which this development would have been impossible; also that Church's calculus of λ-conversion is not a mere variation of the Schönfinkel-Curry calculus of combinators, but represents a different approach. Its relationship to the work of Schönfinkel and Curry is made clear in Church's VI 171(1) (cf. pp. 3-5, 43-51).

Schönfinkel's 3071, written in collaboration with Bernays, is fairly represented as one of the important early papers on the decision problem. The author gives the biographical information that Schönfinkel became mentally ill and died in Moscow in 1942.

Anokhin further summarizes works of Zeigalkin 334t, V 90 (1) (who died on March 28, 1947); Novikov XI 129 (3), XIII 170 (1), XIV 285 (4); Bočvar IV 98 (2), V 119 (1), XI 129 (1, 2), XII 27 (1); Malcev II 84 (2); and Markov XIII 52 (2), XIII 83 (1).

The idea of applying Boolean algebra to the analysis of electrical relay-contact circuits, the author points out, was first put forward in 1910 by the Russian physicist Eréndfés in a review 1961 of the Russian translation (1909/49) of Couturat's L'algèbre de la logique. Eréndfés's proposal for an "algebra of switching circuits" was worked out in detail in 1934-35 by Glivenko's pupil, V. I. Ščastakov, whose results were embodied in a paper written in January 1935. This paper, according to Anokhin, was not published at the time, but formed the basis of Ščastakov's candidate's dissertation, the major part of which was published in 1941 in the journal Tekhniščeskaja fizika (11-6). In the meantime (1938), Shannon had published his paper IV 108 (3) and gained credit for the idea.

This problem was further explored by A. M. Gavrilov in papers published between 1945 and 1947. Ščastakov's later work X11 133 (1), according to Anokhin, is evidence that even many-valued logics have practical significance and technological applications.

In the concluding pages the author summarizes papers by Markov XIV 67 (1) on recur.