Reference 8   English Translation

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Temperature Coefficients of Elastic Constants of Quartz

by

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We have reported in the paper (JIEE Japan, Vol. 54, No. 543, p.940, Oct. 10, 1933 on October 1933) on the thickness vibration of quartz oscillating plates cut in parallel to X-axis (Figure 1) that the vibration mode is sharing vibration and frequency is expressed as

$$f = \frac{q}{2a}\sqrt{\frac{c}{\rho}}, \quad c = \frac{1}{2}(c_{11} - c_{12})\sin^2\theta + c_{44}\cos^2\theta - c_{14}\sin 2\theta \text{ ------- (1)}$$

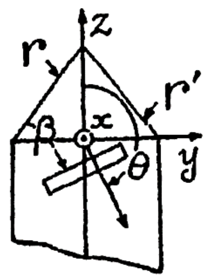

Figure 1

Based on the equation (1), we estimated that temperature coefficients of oscillation would change along with the change of θ and we reported the results of the measurement of temperature coefficients using such plates. Furthermore, we informed that temperature coefficients of adiabatic elastic constants of c_{44}, c_{14} etc. would be calculated and that these results would be presented later. In this paper, we will discuss on this issue.

Variation of frequency f in equation (1) by temperature is given as

$$2\frac{1}{f}\frac{\partial f}{\partial T} = \frac{1}{c}\frac{\partial c}{\partial T} - \frac{1}{\rho}\frac{\partial \rho}{\partial T} - 2\frac{1}{a}\frac{\partial a}{\partial T} \text{ ---------------------------------- (2)}$$

Line expansion coefficients of line of length a with the arbitrary direction (direction cosine l, m, and n) having density ρ is given as

$$-\frac{1}{\rho}\frac{\partial \rho}{\partial T} = \frac{1}{x}\frac{\partial x}{\partial T} + \frac{1}{y}\frac{\partial y}{\partial T} + \frac{1}{z}\frac{\partial z}{\partial T} \text{ ------------------------------------ (3)}$$

$$\frac{1}{a}\frac{\partial a}{\partial T} = l^2\frac{1}{x}\frac{\partial x}{\partial T} + m^2\frac{1}{y}\frac{\partial y}{\partial T} + n^2\frac{1}{z}\frac{\partial z}{\partial T} \text{ ----------------------------- (4)}$$

$$\left.\begin{aligned} \frac{1}{x}\frac{\partial x}{\partial T} &= \frac{1}{y}\frac{\partial y}{\partial T} = 13.7\times10^{-6}/^\circ\mathrm{C} \\ \frac{1}{z}\frac{\partial z}{\partial T} &= 7.5\times10^{-6}/^\circ\mathrm{C} \end{aligned}\right\} \text{ -- (5)}$$

(by G. W. C. Kaye and T. H. Laby : "Physical and Chemical Constants", p.56)

Therefore, equation (2) of temperature coefficient becomes as

$$2\frac{1}{f}\frac{\partial f}{\partial T} = \frac{1}{c}\frac{\partial c}{\partial T} + (7.5 + 12.4\times\cos^2\theta)\times10^{-6} \text{ ------------------------ (6)}$$

By measuring frequencies and temperature coefficients for plates cut at θ = 140° (angle between X-axis and the plate face was within 0.5') and θ = 90° (correspond to Y-cut and angle between X-axis and the plate face was within 0.5'.) are shown in Table 1 and Fig 2.

Table 1

β	θ	dimension	freq.	Temperature coefficient
		mm³	kc	×10⁻⁶/°C
−38° 13'	90° 00'	4.99×21.7×27.0	3 925.6	+103
−38° 13'	90° 00'	5.74×21.1×26.3	3 396.7	+103
−38° 13'	90° 00'	7.16×26.6×26.7	2 727.0	+103
8° 57'	137° 10'	5.41×25.8×29.6	4 661.9	+1.9
9° 31'	137° 44'	5.42×22.2×28.3	4 654.6	+0.6
9° 58'	138° 11'	4.95×25.9×30.1	5 088.5	−0.5
10° 34'	138° 47'	5.42×25.0×29.2	4 653.6	−1.9

Narrow side of the plate is in parallel with X-axis. Frequency measured at 21°C

Temperature coefficients of Y-cut plate increase along with the decrease of the thickness of plates and reach the values listed in Table1.

From Fig 2, at β =9°46' (θ = 137°59') and θ =54°45' (This angle was reported in the paper published on October, 1933) temperature coefficients become zero.

We substitute these values and temperature coefficient 103 X 10^{-6}/°C at θ = 90° into equation (6) as follows,

$$0=\left(\frac{1}{c}\frac{\partial c}{\partial T}\right)_{137°59'}+(7.5+12.4\times\cos^2 137°59')\times10^{-6} \quad \text{------------------------ (7)}$$

$$0=\left(\frac{1}{c}\frac{\partial c}{\partial T}\right)_{54°45'}+(7.5+12.4\times\cos^2 54°45')\times10^{-6} \quad \text{------------------------ (8)}$$

$$2\times103\times10^{-6}=\left(\frac{1}{c}\frac{\partial c}{\partial T}\right)_{90°}+7.5\times10^{-6} \quad \text{------------------------------------ (9)}$$

Then we use following values.

$$\left.\begin{aligned} c_{66}&=\frac{1}{2}(c_{11}-c_{12})=\frac{1}{2}(85.45-7.26)\times10^{10}\ \text{dynes/cm}^2\\ c_{44}&=57.09\times10^{10}\ \text{dynes/cm}^2\\ c_{14}&=-16.87\times10^{10}\ \text{dynes/cm}^2\end{aligned}\right\} \quad \text{-------------------- (10)}$$

From equations (9), (1) and (10),

$$\left.\begin{aligned} \left(\frac{1}{c}\frac{\partial c}{\partial T}\right)_{90°}=\frac{1}{c_{66}}\frac{\partial c_{66}}{\partial T}&=+199\times10^{-6}\\ \frac{\partial c_{66}}{\partial T}&=+77.8\times10^{6}\end{aligned}\right\} \quad \text{----- (11)}$$

(+77.8X10^{-6} in the original Japanese text is typo)

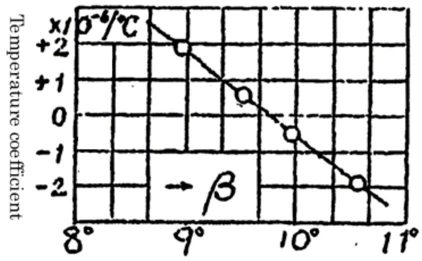

Figure 2

It is obvious that

$$\frac{\partial c}{\partial T}=\sin^2\theta\frac{\partial c_{66}}{\partial T}+\cos^2\theta\frac{\partial c_{44}}{\partial T}+\sin 2\theta\frac{\partial c_{14}}{\partial T} \quad \text{---- (12)}$$

From equations (7), (8), (10), (11) and (12)

$$\frac{\partial c_{44}}{\partial T}=-113.5\times10^{6},\quad \frac{1}{c_{44}}\frac{\partial c_{44}}{\partial T}=-199\times10^{-6} \quad \text{----- (13)}$$

$$\frac{\partial c_{14}}{\partial T}=-18.5\times10^{6},\quad \frac{1}{c_{14}}\frac{\partial c_{14}}{\partial T}=+110\times10^{-6} \quad \text{-------- (14)}$$

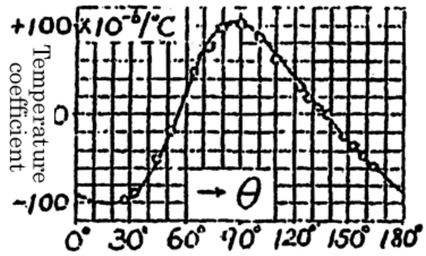

Figure 3

Once we obtain temperature coefficients of adiabatic eristic constants, we can derive temperature coefficients of frequency using equations (1), (6), (11), (13) and (14). Figure3 indicate the results of the calculation. Several small circles around the curve in Figure 3 correspond to the measured values of temperature coefficients which we reported in the previous paper on October 1933. The curve in Figure 3 shows good agreement with the experimental data.

Additionally, Temperature coefficient c_{11} in equation $c_{60} = \frac{1}{2}(c_{11} - c_{12})$ can be calculated by measuring temperature coefficient of frequency of X-cut plate. Then we can get temperature coefficient of c_{12}. We will report these in the next time.

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