Reference 11 Abridged English translation (for figures and equations, refer to the original paper)

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Vibration of Thin Piezoelectric Quartz Plates
(Especially on R_1 -cut Rectangular Plates)

by

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1. Introduction

After the discovery of oscillation by quartz plates, thickness vibration of thin plates have been widely used in high frequency fields. However, these plates have disadvantage that the proximate vibrations exist near the main frequency. In recent year, one of authors established a detection system of electric polarization on plates by using a small needle. Using this detector, we could make clear the precise behavior of vibrations and modes. In this article, we intend to explain about the application of this system to R_1-cut plates and obtained results.

2. Relation between Thickness Vibration and Dimensions of Thin Plates

Here we assume y axis as the direction of the normal line of R_1 plate as shown in Fig 1. When the area of plate is infinitely large and angular frequency ω of oscillating vibration is calculate as,

$$\omega\ y_0\sqrt{\rho / C66} = q\ \pi \quad (q : \text{positive integer}) \text{ --------- (1)}$$

$$u = A \cos\ (q\ \pi\ y / y_0)\exp(j\ \omega\ t) \text{ ---------------------- (2)}$$

where, p is density, C_{66} is elastic constant and u is displacement in x (parallel to surface plane) direction.

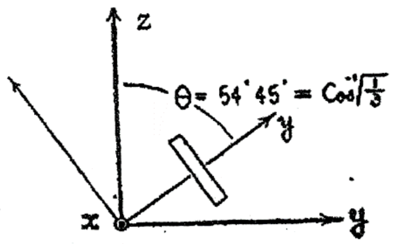

Figure 1 R_1 Plate

In infinitely wide plate, only above vibration can be exited and other two modes cannot be exited. However, in finite-size plates, many oscillation modes exist.

For a finite size plate, in order to investigate on existing vibrations and frequency deviations from formula (1) of principal wave, only simple detector circuit can be available now. But conventional simple method cannot make clear the detail of behavior.

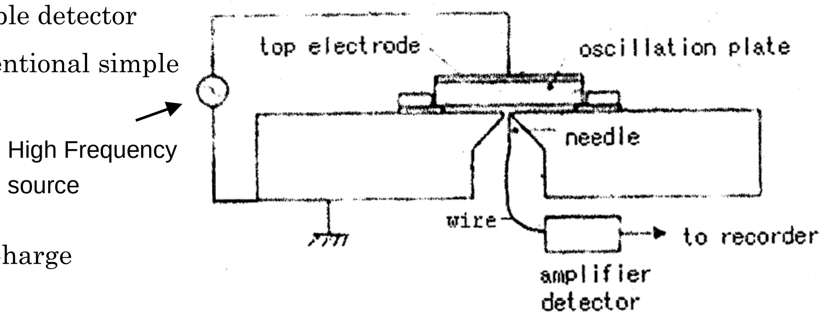

Figure 3 New Detector using Needle

3. Observation of Polarization Charge Distribution by Detection Needle

One of authors proposed a detector as shown in Fig 3. Detecting needle (0.3 mm radius) is inserted from down side hole of a movable plate. By testing with well-known existing plate (example, X-cut), this measurement system showed satisfactory result for obtaining (sinusoidal) distribution of polarization. (Fig. 4).

4. Measurement of Oscillation Frequency

Frequencies of various vibrations exiting on the plate can be measured with the system by utilizing vacuum-tube synchronization circuits combined with recorder (Fig 5). Amplitude of tunable oscillator output drops when frequency is synchronized with that of plate vibration.

5. Thickness Vibration (l)

To obtain final expression and formula for modes on the plate, theory of plane wave expressions of thin plate and their solutions are introduced. [Equations (3) ~ (7)]. As the final result, displacement u of vibration is obtained as sinusoidal form, and polarization on the surface of a plate along x and z axis is expressed as

$A \sin (p \pi x / x0) \cos (r \pi z/z_0)$, (p and r are harmonic integers)

Obtained typical polarization mode distributions are shown in Fig. 6. Fig. 7 indicates relation between frequencies of observed oscillations and thickness of plates. In Fig. 8, measured polarization distribution on the plates in x- and z-direction of crystal are shown. These figures indicate that the actual vibrations are a mixture of principal and asymptotic modes.

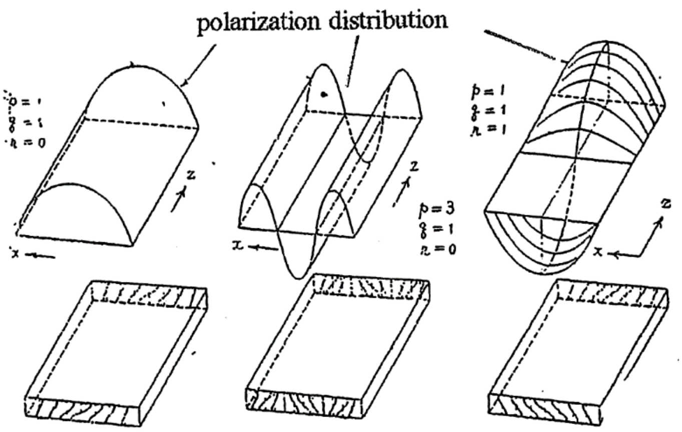

Figure 6

6. Higher-Order Edge Vibrations (1)

We intend to discuss on the vibrations having frequency lower than main oscillation mode. These vibrations may be thought in three cases as follows,

(1) Resonant frequency is dependent on only one dimensional size x_0, and independent from other two sizes.

(2) Dependent only on z_o,

(3) Dependent on x_o and y_o, but independent from z_o.

As shown in Fig 9, above properties are certified for various plates cut along with several surfaces of crystal. Furthermore we can get frequency expression formulas (8) ~ (11) for above three cases. From these formulas and measured frequencies using our detector, it became possible to get elastic constants of crystal for various orientations.

7. Thickness Vibrations (2)

As discussed in Chapter 2, frequency of main vibration is expressed by formula (7). But in finite dimension plates, calculated frequencies by this formula have some deviation from these measured by our detector. In this chapter, we discussed on the deviation of these frequencies and found ways for correction of these deviation by modifying elastic constants.

8. Higher-Order Edge Vibrations (2)

We can find in Fig.9 other vibrations that are not discussed in previous chapters. An example of this mode is shown in Fig 10 and 11. The reason of existence of this mode may be thought by the edge effect that induced electric polarization charges at the end of plate is reduced. Flexural vibration called in US papers [(1), (2), (4)] may be thought as this type of mode. We clarified this time by our measurements.

9. Coupling of Thickness Vibrations and Higher-Order Edge Vibrations

By observing the measured results of Fig 13, we can conclude as,

(a) Thickness vibrations except main one are coupled with edge vibrations

(b) Higher-order edge vibrations are less exited in higher frequency region than main mode

(c) Coupling of thickness vibration and higher-mode of edge vibrations become stronger for higher harmonic number.

10 Conclusion

Conclusions can be summarized as follows.

(a) In finite-size vibration plate, many modes of vibration can be exited.

(b) We could certify high utility of our detection system for measuring polarization charge on the plate.

(c) By applying our detection system to R_I-cut plate measurement, following results are obtained,

(d) Frequency of main vibration (thickness mode) is independent on the size in z direction, and slightly increases by decreasing x direction size

(e) (f) (g) We could clarify on the behavior of proximate vibrations which were excited by coupling of principal vibrations and higher-order edge vibrations

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