Draft Paper: Abacus Division as Alternative to Babylonian "Technique"
by Stephen Kent Stephenson, sks23@cornell.edu Saved August 30, 2015 at 8:43 AM

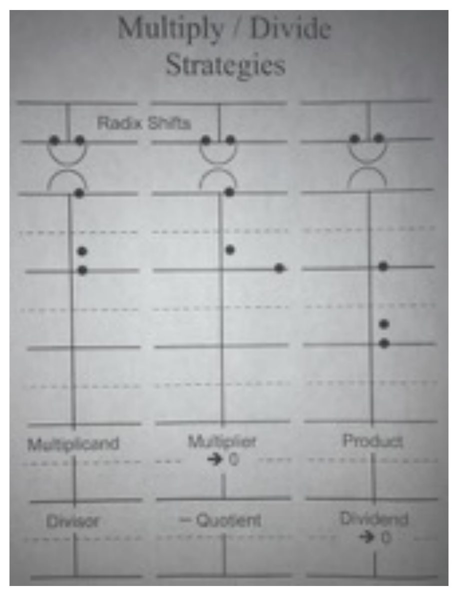
In response to Section 13 of:

Babylonian Mathematical Texts I. Reciprocals of Regular Sexagesimal Numbers
Author: A. J. Sachs; Source: Journal of Cuneiform Studies, Vol. 1, No. 3 (1947), pp. 219-240
Published by: The American Schools of Oriental Research
Stable URL: http://www.jstor.org/stable/1359434; Accessed: 06-08-2015 15:56 UTC

In Section 13 Sachs demonstrates the use of "The Technique" for finding the reciprocal of a regular sexagesimal number c = 5,3,24,26;40 and notes that **"The Technique" will not work for numbers that are not regular**. ("Regular" means the number's reciprocal is a terminating number. In their "decimal" form all rational numbers either terminate or repeat.) There are more non-regular numbers than regular!

If by chance you want to divide by a number that is not regular, you could waste a lot of time trying "The Technique" without success; esp. since without paper and pen you'd have to use clay to make tablets, reeds, and a bronze knife to whittle the reeds into styluses to write on the clay tablets you make. Why not do the division directly on a three table line abacus instead?

As an example, let's calculate 1/c = 1/5,3,24,26;40 using an abacus; harvesting quotient digits as they're formed; scaling the remaining quotient and remainder by shifting up in order to get as many exact digits as we want (until termination in this example).



Per the strategy graphic to the right, we'll put the divisor in the left table, the dividend in the right table, and form the negative quotient in the middle table.

Below is the initial setup. Each of the left, middle, and right tables have both a top and bottom section; the top for radix shift counts (exponents of 60) and the bottom for fractions of one. Horizontal lines above and below the Title are unit lines.

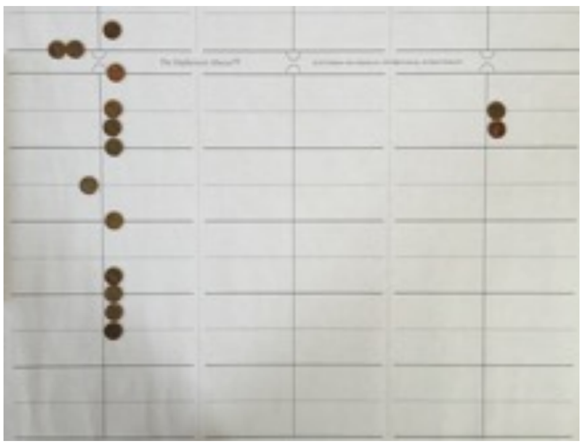
The vertical bars separate negative from positive (negative on left of vertical bar, positive on right). The divisor reads (;5,3,24,26,40)x60^4 and the dividend is 1.

At right we’ve halved both divisor and dividend twice.

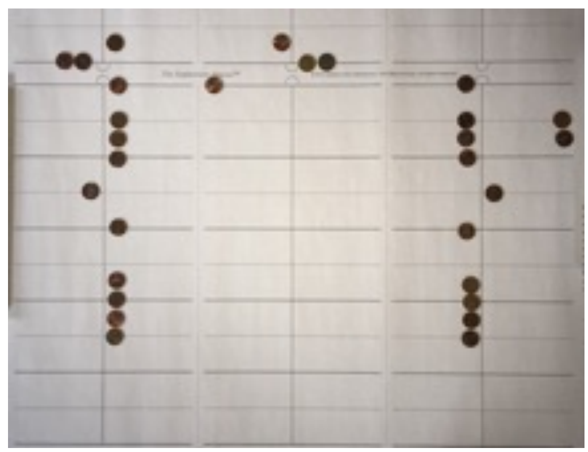
(;5,3,24,26,40) x 60^4) / 2 / 2
= (;1,15,51,6,40) x 60^4

Below we’ve shifted the divisor up one digit.

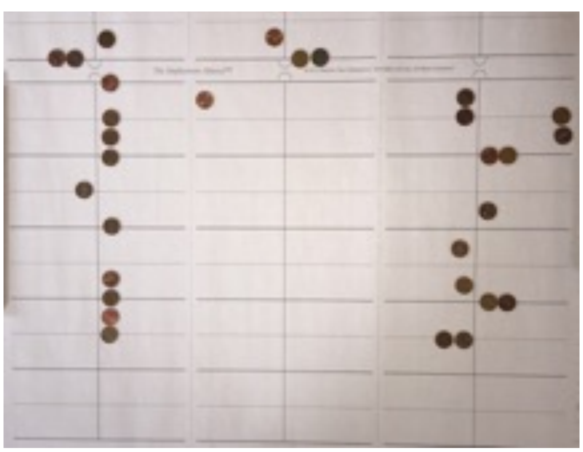
(;1,15,51,6,40) x 60^4 = (1;15,51,6,40) x 60^3



Below we’ve entered the quotient exponent as –3 = 0 – 3. We’ve also entered –1; in the negative quotient table and a negative copy of the divisor in the bar# of the dividend table. Each table holds two numbers: a bar# next to the vertical bar and an edge# on the edges. The edge# is currently the stored dividend remainder, ;15.



Below we’ve halved the quotient edge# and the dividend bar# as too big compared to the edge# stored remainder.

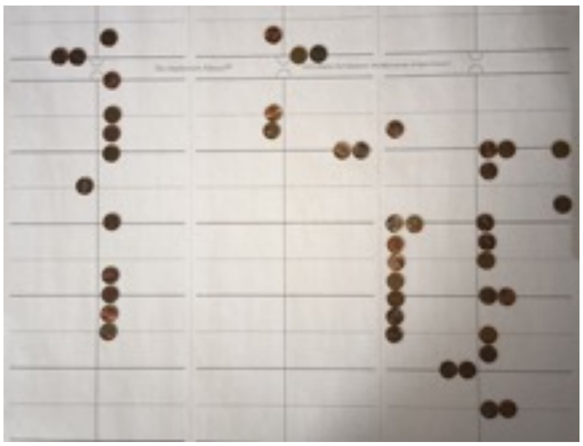


At right we halve again and make readable:
–;18,57,46,40.

Dividend bar# and edge# combined into new edge#.

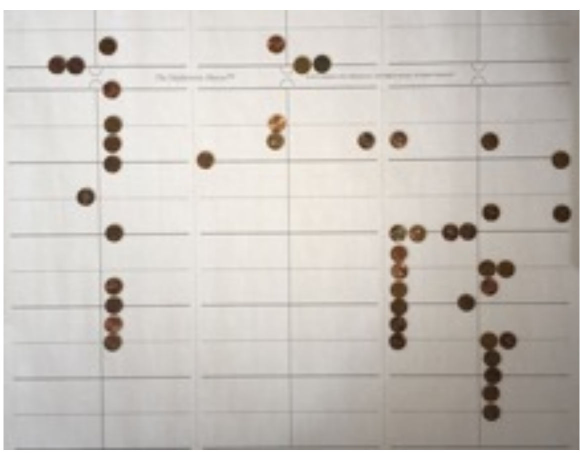
Positive copy of divisor x (;1) entered as bar# and doubled.

Dividend bar# made readable below.



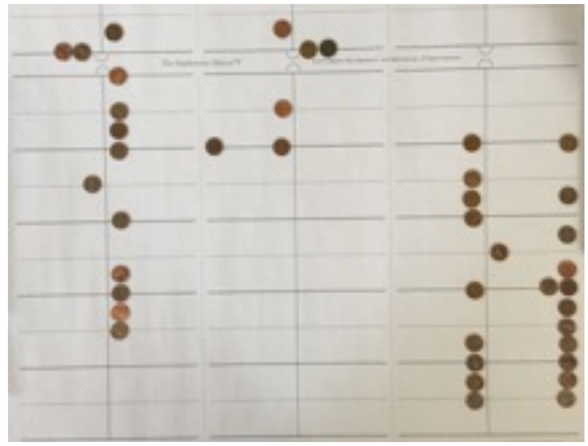
Dividend bar# doubled below.

Dividend bar# made readable below.



Dividend bar# and edge# combined into new edge# below.

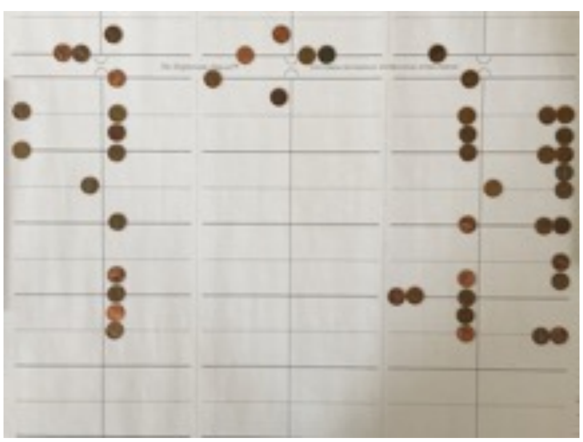
Dividend edge# made readable and negative copy of divisor x (;1) entered as bar# at right.



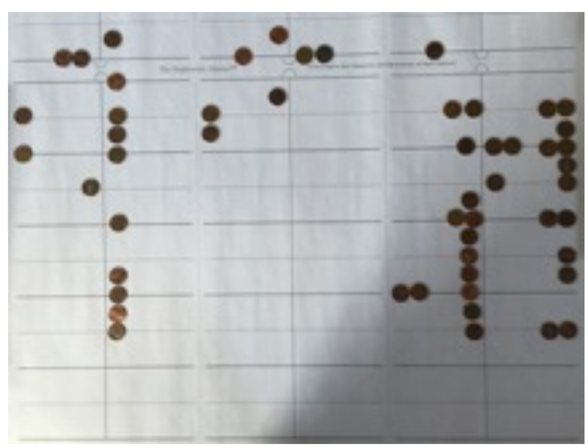
Below, dividend bar# is halved.

Dividend bar# and edge# combined into new edge# below.

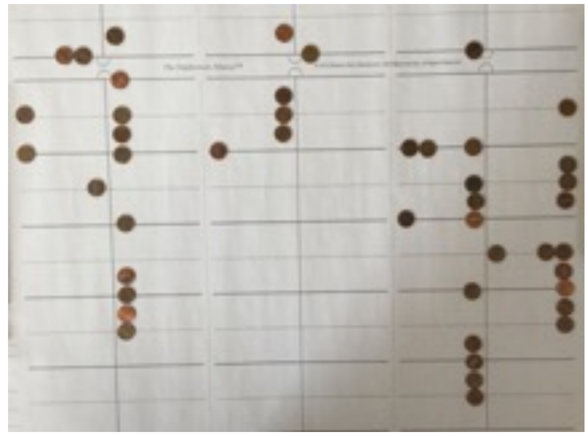
Below, first digit of quotient harvested and stored in left edge of divisor table; quotient and dividend shifted up one digit; and negative copy of divisor entered in dividend bar#.



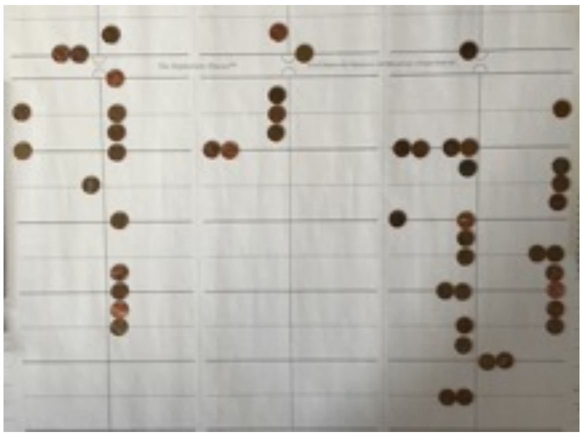
Below, dividend bar# halved twice.



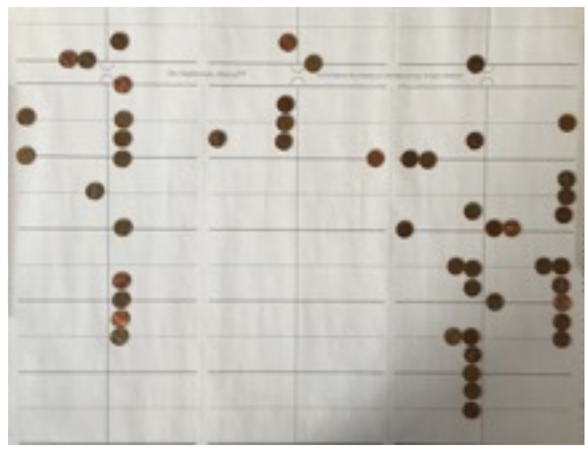
At right, dividend bar# and edge# combined into new edge#; negative copy of divisor entered into bar#, second line down.



Below, dividend bar# doubled.

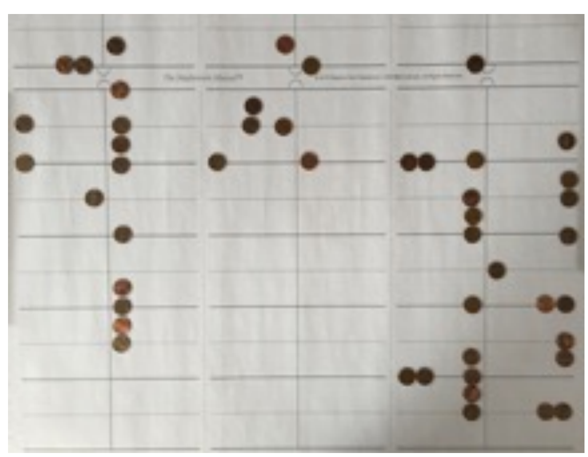


Below, dividend bar# doubled again.

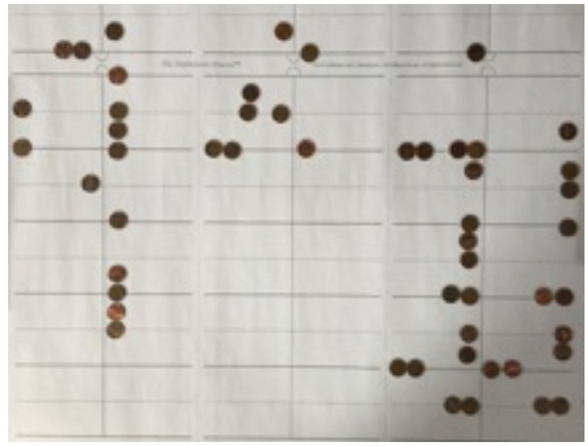


Below, dividend bar# and edge# combined into new edge#.

Below, negative copy of divisor entered into dividend bar#, second line down.

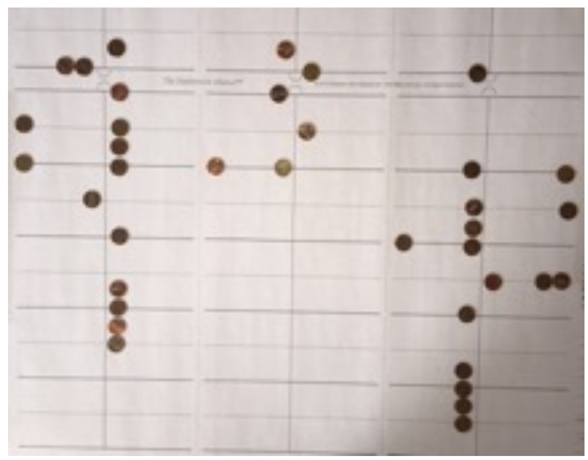


At right, dividend bar# doubled.



Below, a combining of dividend bar# and edge# into new edge#.

Below, negative copy of divisor entered into dividend bar#, second line down.



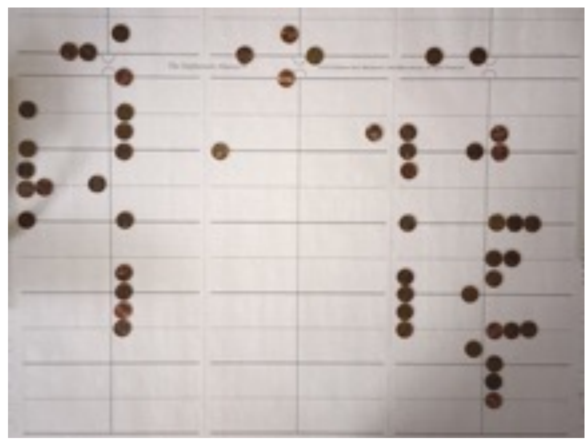
Below, a combining of dividend bar# and edge# into new edge#.

Below, second digit of quotient harvested and stored in left edge of divisor table .

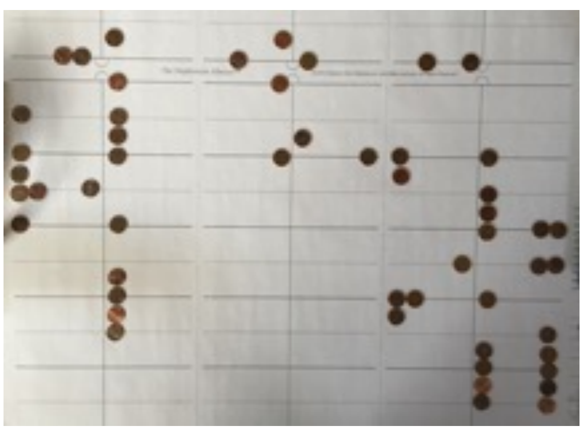
At right, quotient and dividend remainder shifted up one digit; a negative one added to each of their exponents.

Below, a positive copy of divisor added to dividend bar# and doubled.

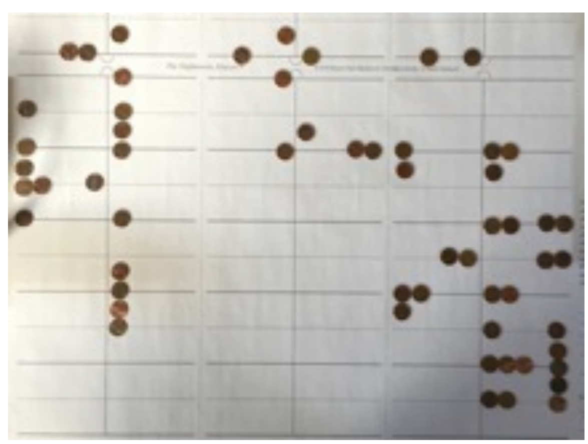
Below, dividend bar# doubled again.



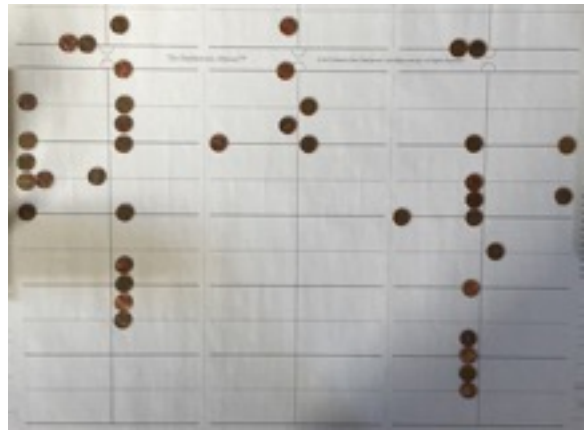
Below, dividend bar# and edge# combined into new edge# and a positive copy of divisor entered as new bar# one digit down.



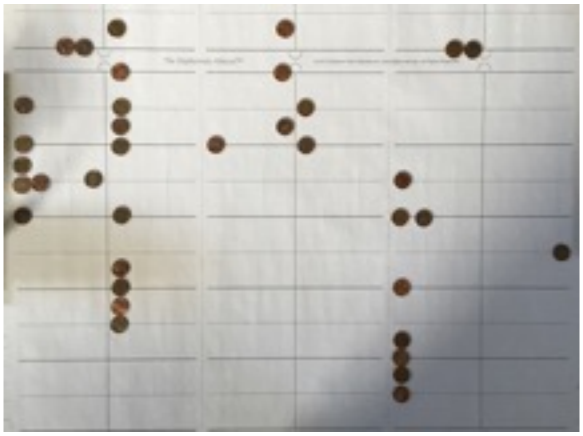
Below, dividend bar# doubled.



At right, dividend bar# added into edge#; negative copy of divisor entered as new bar# one digit down.

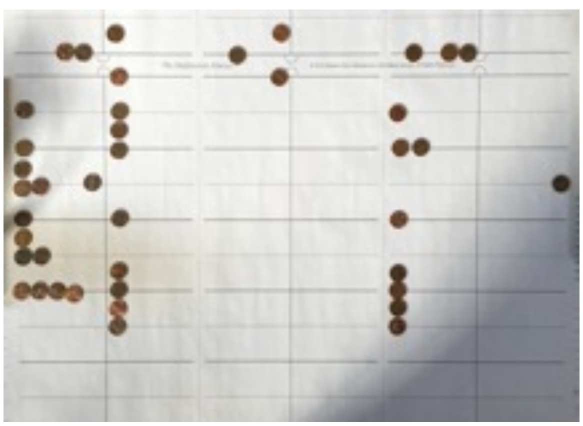


Below, dividend bar# and edge# have been combined into new edge#.



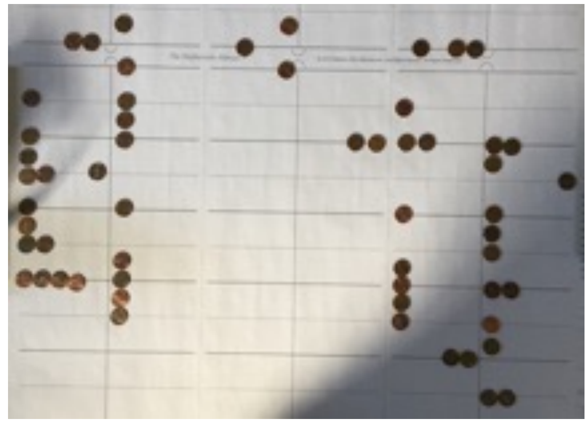
Below, we've harvested and stored the third quotient digit. It's 54 not 55 because you can see that we will be adding some multiple of the divisor into the second digit of the dividend.

Below, the remaining quotient and dividend have been shifted up one digit, with a negative one added to both of their radix shifts (exponents).

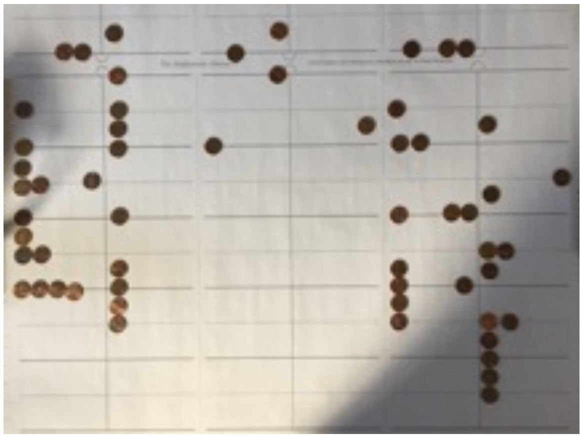


Below, a positive copy of the divisor has been entered into the dividend bar# one digit down.

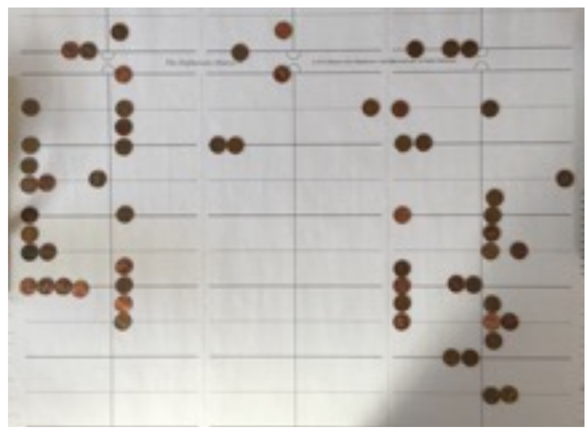
At right, dividend bar# has been doubled.



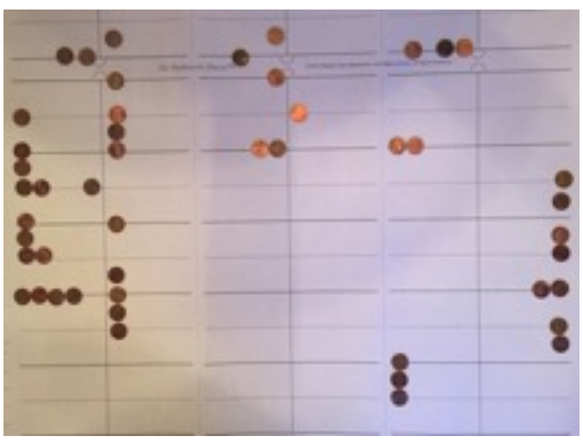
Below, dividend bar# has been doubled again.



Below, dividend bar# has been doubled again.

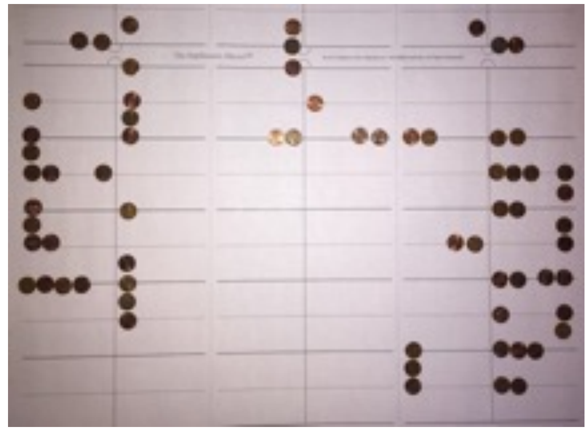


Below, dividend bar# and edge# have been combined into new edge#.

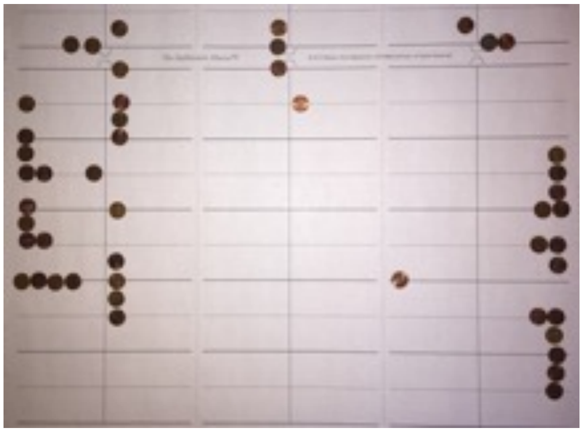


Below, a positive copy of divisor has been entered into dividend, one digit down.

At right, dividend bar# has been doubled.



Below, dividend bar# and edge# have been combined into new edge#.

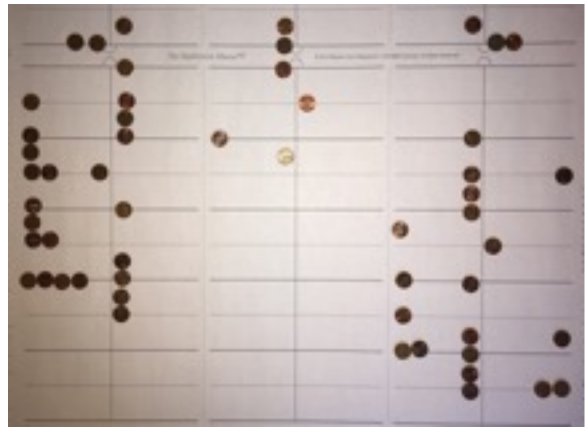


Below, dividend bar# has been halved.

Below, a negative copy of divisor has been entered into dividend bar#.

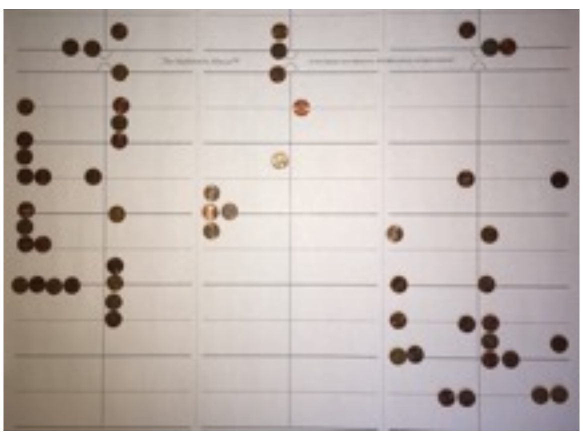
Below, dividend bar# and edge# have been combined as new edge#.

At right, a negative copy of divisor is entered as new dividend bar#, one digit down.



Below, dividend bar# has been halved.

Below, dividend bar# has been halved again.



Below, dividend bar# has been halved again.

Below, dividend bar# and edge# have been combined with zero left in the dividend remainder; so we’re done.

The quotient table now reads – (;50,37,30)x60^(–6), so the partial quotient is ;0,0,0,0,0,0,50,37,30. Adding the saved digits our final answer is:

;0,0,0,11,51,54,50,37,30 = 1/c = 1/(5,3,24,26;40).

AND this method can be used to divide by numbers that are NOT regular (more than half the time?)!