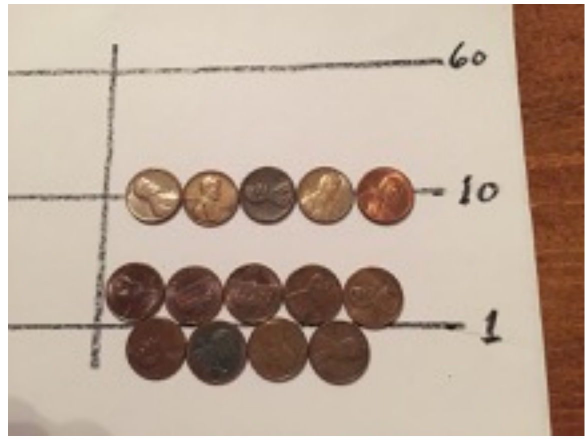
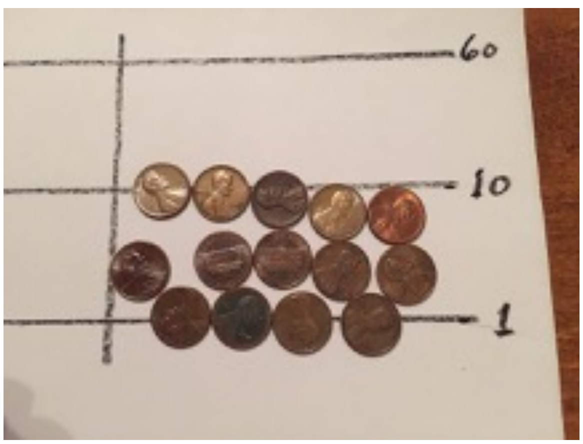
Draft Paper: 59's Abacus Representations Over Time;
and Playfully Counting By 11's.
by Stephen Kent Stephenson, sks23@cornell.edu Saved September 10, 2015 at 9:25 AM

Let's use the one digit sexagesimal number 59 to explore the possible evolution of Babylonian abaci from the 4th millennium B.C.E.

The first abaci could have had tokens / counters placed only on the unit line and 10 line of each digit. 59 would have 5 tokens on the 10 line and 9 tokens on the 1 line (picture at right).



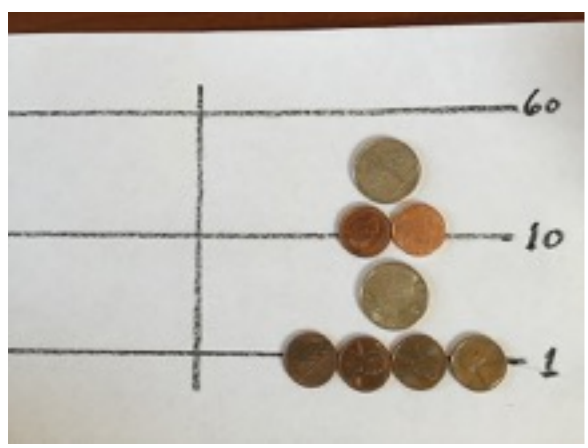
Whups; Achmed, my classmate, just came by and bumped my counting board (below). Sometimes when this happens the tokens are even more jumbled up; this time it shouldn't be too hard to determine their proper location.



So that's how errors of misinterpreted 1 and 10 counters could have been generated; by accident, not by errors in math.

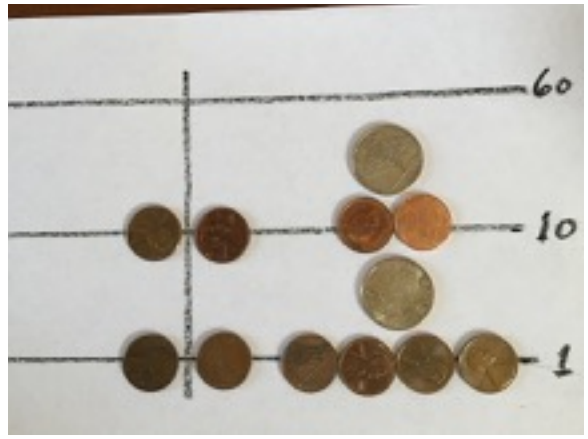
The scribes would certainly discover this problem and realize it was a function of the large number of tokens on the board, the close proximity of the 1 and 10 tokens, and the fact that the 1 and 10 tokens look the same.

The solution they could have found would have reduced the number of tokens and separated the 1 and 10 tokens with a new different looking token, the 1/2 token. For 59 they would have replaced the 5 tokens in the top row on the unit line with a 1/2 token (1/2 the value of a token on the line above) in the space between the 1 line and the 10 line (right). For symmetry they would have also replaced 3 ten tokens with a 1/2 token in the space above them. Now it's very clear where the 1 and 10 tokens are and the board is much less prone to misreading if it's jostled a bit.

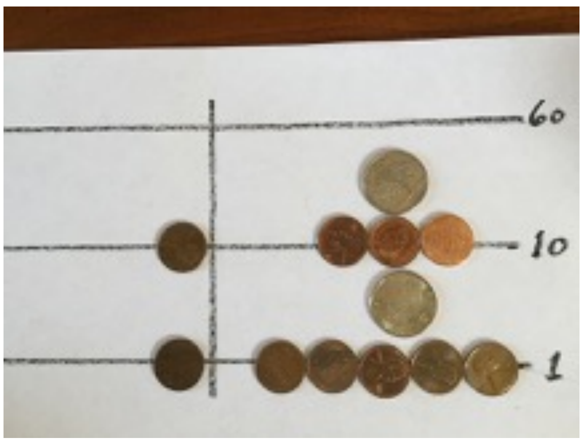


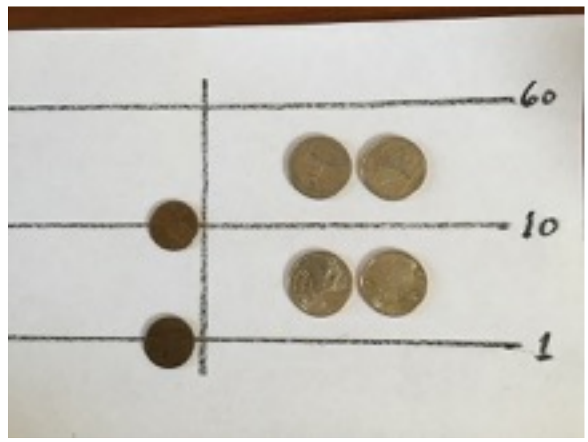
The area to the left of the vertical bar would be used to enter a number you want to subtract from the number on the right of the vertical bar. We'll use it next as we play with the abacus.

One playful / exploratory thing we could do is add and subtract 11 (11 affects both the top and bottom parts of the number) to see what interesting patterns of tokens we might find (right). Since we’ve added zero (11–11) we haven’t changed the value on the board.

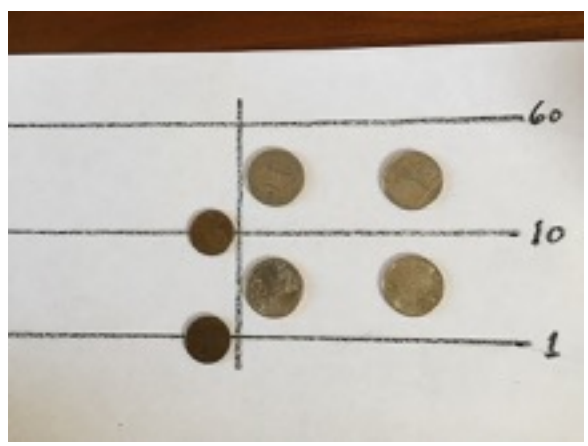


Rearranging the right side of the board (below) we see we can promote both the 1 and 10 line tokens (right).

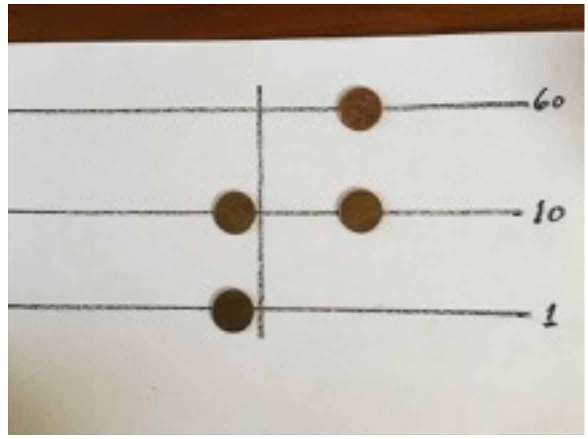




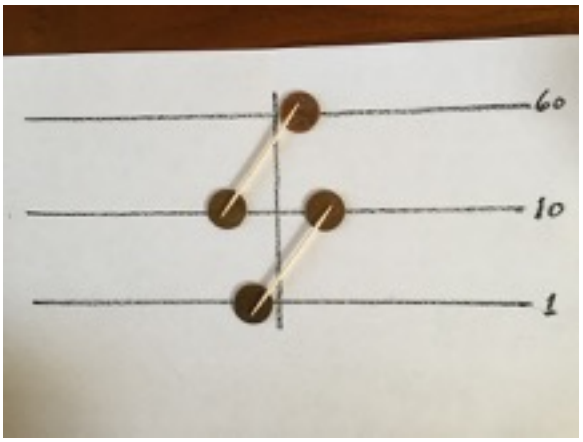
Rearranging slightly we get the configuration at the right. Two combinations may prove useful in the future: 30–10=20 and 5–1=4. 20 takes two tokens either way (10+10, 30–10) but 30–10 has one token on a space and the other on a line; that might save board space in the future. 5–1 saves two tokens and has one token on a space and a line.



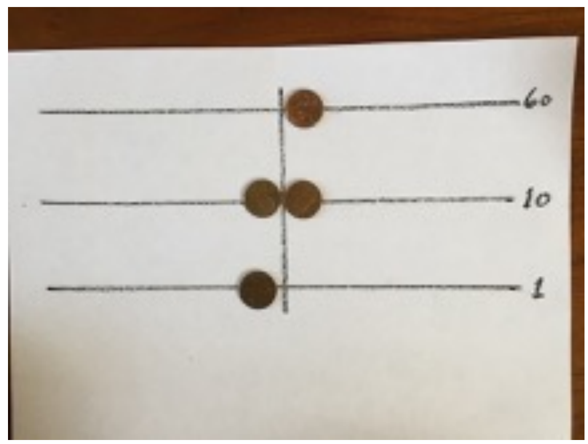
Let’s promote the 1/2 tokens (right) and rearrange (below).



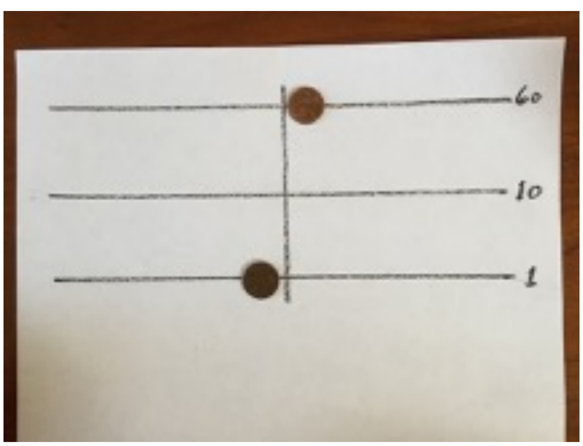
Two new interesting combinations appear: 60–10=50 and 10–1=9; both save at least one token over previous representations.



Rearranging a bit, we see a zero pair (below);



which we can remove without changing the value on the board (below). Another possibly useful configuration appears: 60–1=59 using only 2 tokens.



Not yet found, but easily understood are: 5–2 = 3 and 10–2 = 8. Both save line space/length by changing 3 on a line to 2 on a line.

Hilprecht's 20–1 = 19 and 20–2 = 18 would be entered as (10–1) + 10 and (10–2) + 10.

Using these shorthand configurations an abacus would have far fewer tokens on it, so we can probably dispense with the different size/shape 1/2 token and just use a regular token for 1/2 from now on. However, like training wheels on a bike, new abacists may use different size/shape 1/2 tokens until they learn to “balance”.

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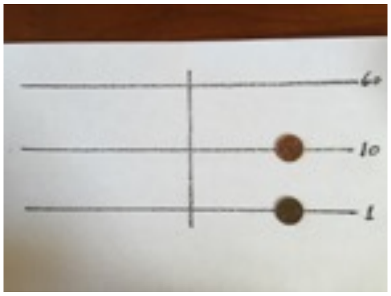
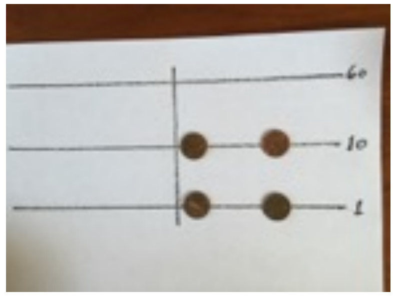
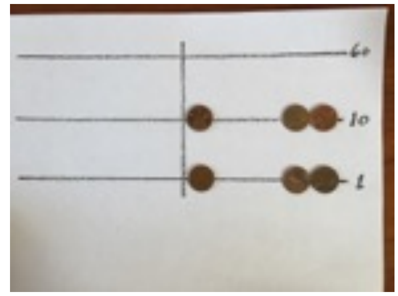
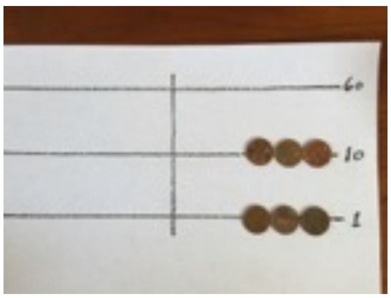
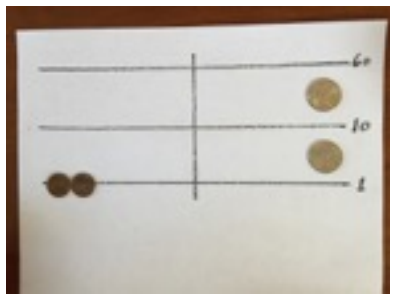
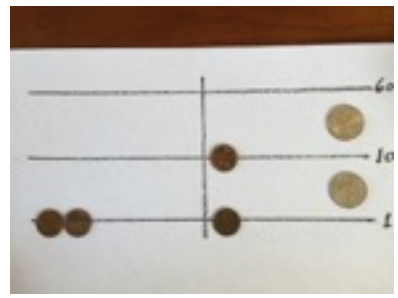
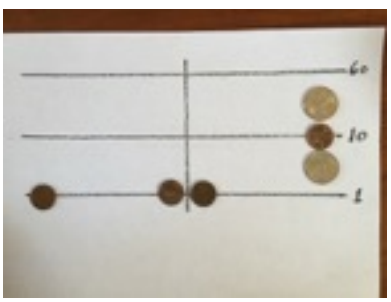
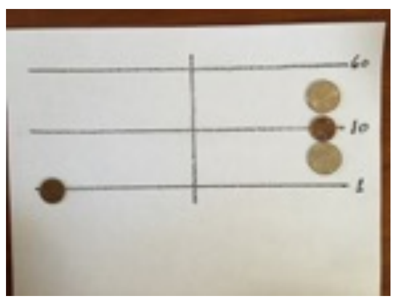
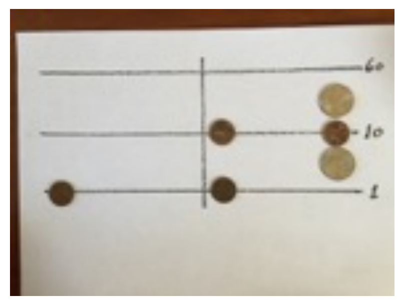
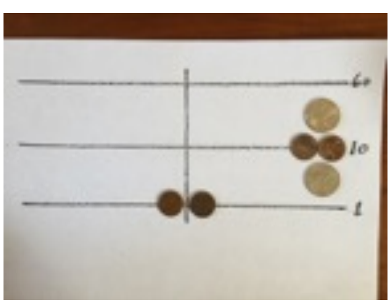
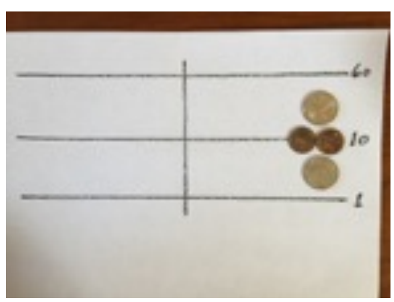
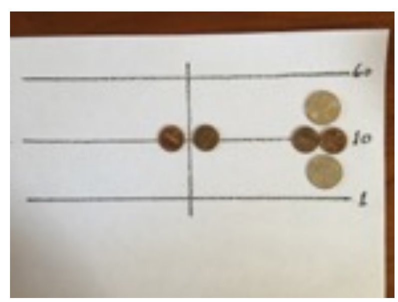
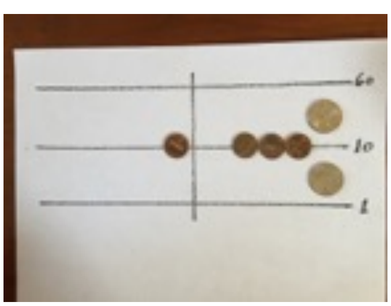
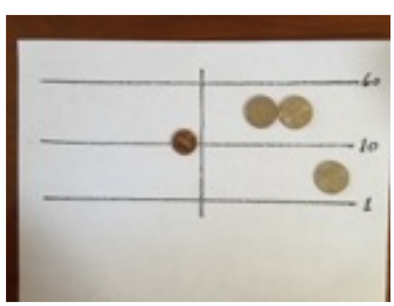
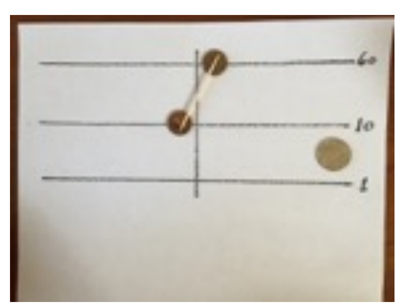
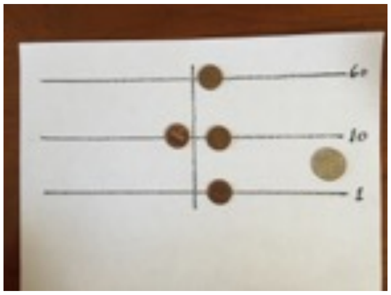
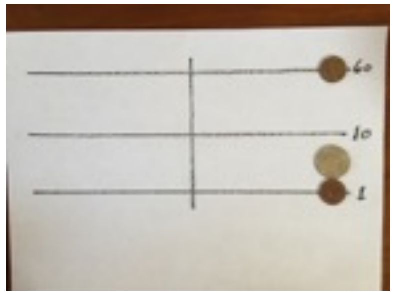
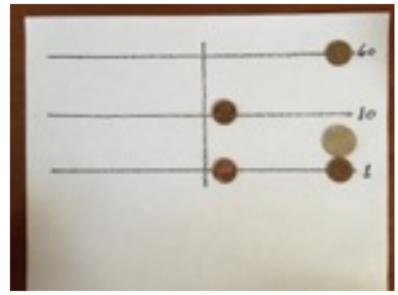
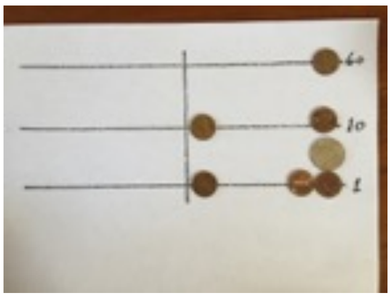
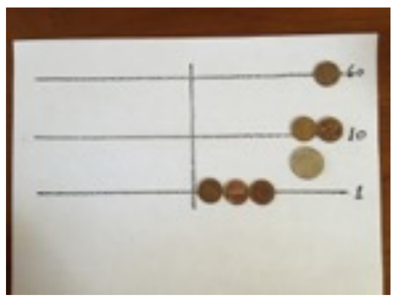
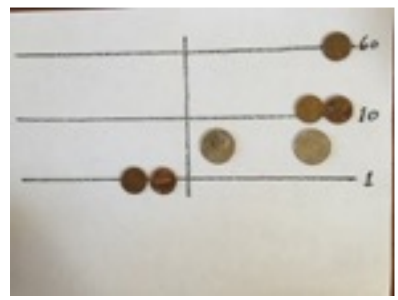
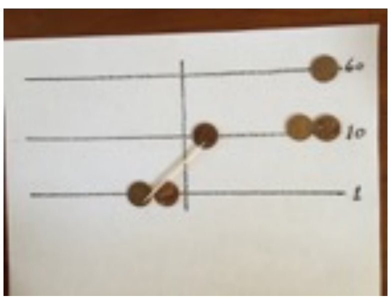
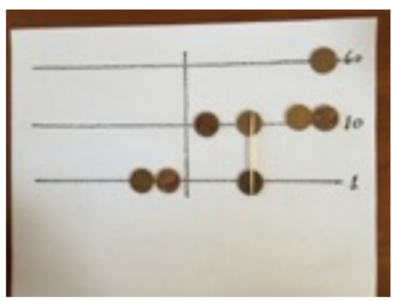
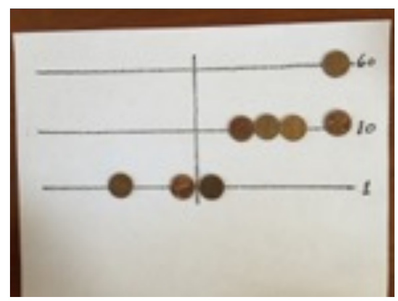
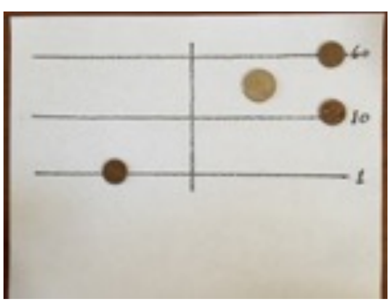
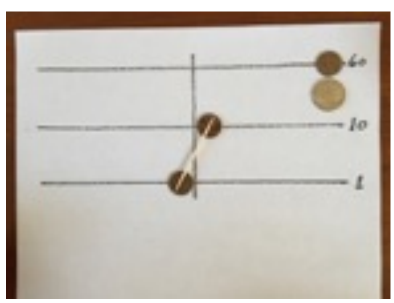
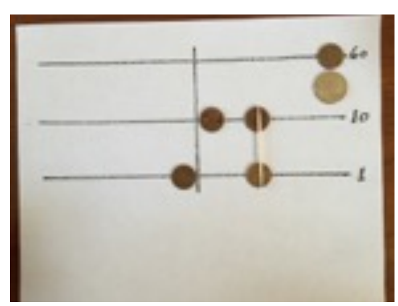
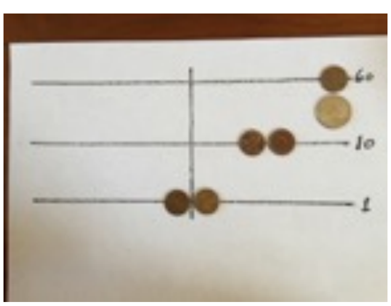
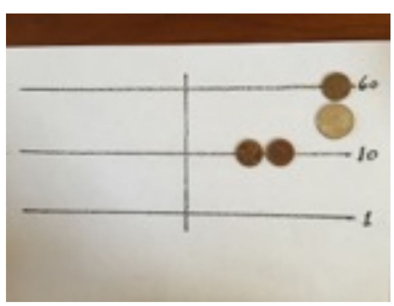
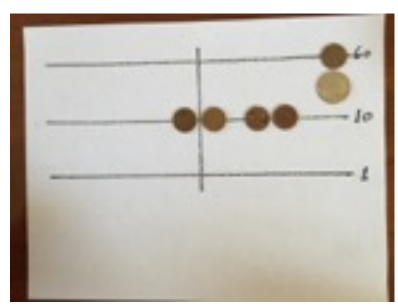
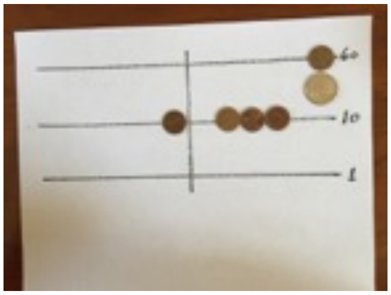
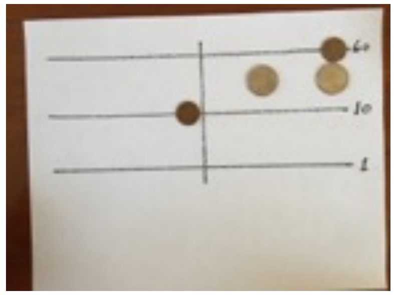
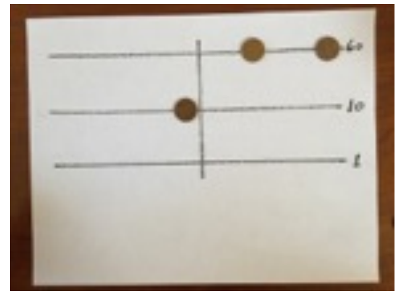
Since playing with our abacus by adding & subtracting 11 from 59 proved to be so interesting, for our next play activity let’s start with 11 and repeatedly add 11.

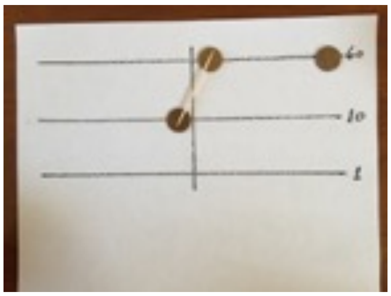
























1,50; = 110.

What time period are we in at this stage of abacus development?

From Melville's Chronology (http://it.stlawu.edu/%7Edmelvill/mesomath/chronology.html):

> 2100-2000 BCE, Ur III or Neo-Sumerian, Development of sexagesimal place value notation.
>
> 2000-1600 BCE, Old Babylonian, Main source of knowledge of Mesopotamian mathematics. Tables: multiplication, roots, powers, reciprocals, coefficients. Algorithmic problems.

So we're probably in the beginning of the Ur III period at the end of the 3rd millennium B.C.E.

I suspect that few to no texts describing these abaci manipulations have been found. But I also suspect that few to no texts describing ***how*** to build a brick wall have been found. But the Babylonians certainly built brick walls: https://en.wikipedia.org/wiki/Architecture_of_Mesopotamia. In both cases the knowledge was transmitted verbally and by demonstration (wall, abacus); only the results exist (walls and cuneiform sexagesimal number tablets).