Synchrophasors for State Estimation in Power Systems with Bad Data Elimination

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Abstract

Due to the increasing concern about environmental factors, the reliability of the entire system, and service quality, power grids in many countries are undergoing a revolution towards a more distributed and flexible "smart grid." With this vision, the traditional supervisory control and data acquisition (SCADA) systems are far from sufficient for the new smart grid. Among the various tasks in the SCADA system the power system state estimate (SE) is a very crucial one. However, conventionally only the real and reactive power measurements are available, and the state estimate can only be obtained from an iterative method, which is computationally expensive and subject to long delays. Fortunately, with synchronization from Global Positioning System (GPS) satellite signals, direct, accurate and synchronized measurements of the voltage and current phasors in the power system become available via phasor measurement units (PMU). With these synchrophasor measurements, real-time monitoring of the system state becomes possible and an entirely new SCADA system for smart grid can be developed. In this project, IEEE Std C37.118.1-2011 [1] will be briefly introduced, and the synchrophasor measurements defined in this standard will then be used for state estimation in power systems with possible bad measurements specifically considered.

I. Introduction

With synchronization from Global Positioning System (GPS) satellite signals, direct, accurate and synchronized measurement of the voltage and current phasors in the power system become available from phasor measurement units (PMU) [2]. With the smart grid increasingly demanding more powerful supervisory control and data acquisition (SCADA), the synchronized phasor measurements provided by PMUs can be beneficial to the system state estimation (SE) process [3]. In conventional power systems, only the real and reactive power measurements are available, and the state estimate can only be obtained from an iterative manner [4]. With PMU measurements, the state estimate can be obtained linearly [5]. With these potential applications of synchrophasor measurements in mind, PMU devices are developed and the standard for PMU devices is specified in IEEE Std C37.118.1-2011 [1].

However, while PMU measurements are expected to significantly improve SE accuracy, there is very limited literature on SE from PMU measurements with bad data. Many just inherit the largest residual removal (LRR) method in conventional SE (see e.g. [6, Chapter 7]). However, with the linear signal model enabled by PMU measurements more sophisticated algorithms become possible.

In this project, we develop and analyze several algorithms by explicitly incorporating the bad data into our system model. To deal with the bad data, one can either estimate the locations first, or estimate the locations and values simultaneously. With the former approach, the determined bad data locations can be used directly to simply remove the contaminated measurements, or they can facilitate the estimation of the bad data values and subtract them from the measurements. We show that these will result in identical state estimators. Thus, bad data removal is actually a simplified form of bad data subtraction with separate location and value estimations.

II. System Model Under IEEE Std C37.118.1-2011

With PMU measurements, the signal model is given by:

$$\boldsymbol{m} = \begin{bmatrix} \boldsymbol{m}_v \\ \boldsymbol{m}_i \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} \\ \boldsymbol{Y} \end{bmatrix} \boldsymbol{s} + \boldsymbol{e} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{b} + \boldsymbol{\eta} \tag{1}$$

where $\boldsymbol{m}_v$ and $\boldsymbol{m}_i$ are p voltage and $q-p$ current synchronized phasor measurements, respectively; $\boldsymbol{s}$ is the state of the power grid, which contains the p bus voltage phasors; $\boldsymbol{Y}$ is the admittance matrix, which is determined by the power grid structure and the transmission line parameters [9, Chapter 1]; and $\boldsymbol{e}$ is the measurement error vector, composed of the measurement device noise, $\boldsymbol{\eta}$, and possible high-magnitude bad data, $\boldsymbol{b}$, due to communication errors or equipment failures. Since communication errors or equipment failures are rare, we expect $\boldsymbol{b}$ to be a sparse vector, i.e., $\|\boldsymbol{b}\|_0 \ll p$.

Under IEEE Std C37.118.1-2011, the synchrophasor measurement accuracy is defined as the *total vector error* (TVE) as follows:

$$\text{TVE} = \sqrt{\frac{\|m - m_{\text{true}}\|^2}{\|m_{\text{true}}\|^2}}\ ,$$

where m is the complex phasor from the measuring device and m_{true} is the true complex phasor value. This error is represented as $\boldsymbol{\eta}$ in Eq. (1). By the law of large numbers, we can model the entries of $\boldsymbol{\eta}$ following the i.i.d. proper complex Gaussian distribution with variance σ^2, where $\sigma^2 = \text{TVE} \times \|\boldsymbol{m}\|^2$.

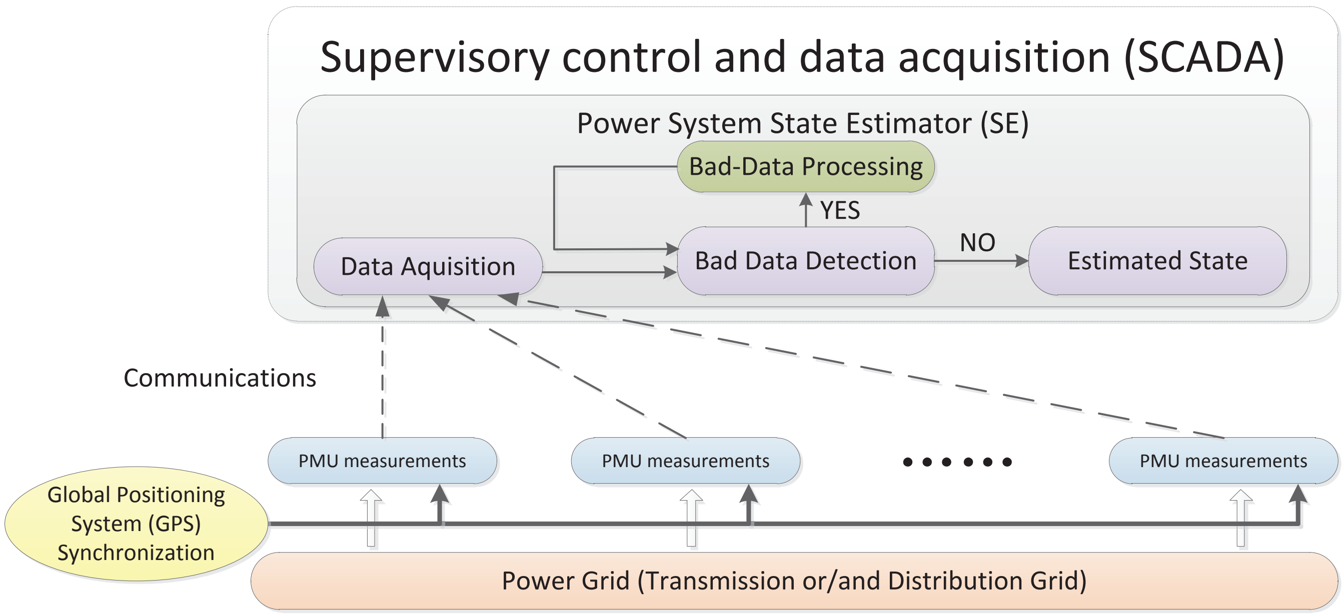

Fig. 1. SCADA with synchrophasor meausrements.

The system structure is shown in Fig. 1. First, assuming the absence of bad data, $\boldsymbol{b}$, the maximum likelihood estimator of the system state is the least-squares (LS) estimator [5]:

$$\hat{\boldsymbol{s}}^{ML} = \hat{\boldsymbol{s}}^{LS} = (\boldsymbol{H}'\boldsymbol{H})^{-1}\boldsymbol{H}'\boldsymbol{m}. \tag{2}$$

Then, the measurement residual is

$$\boldsymbol{r} = \boldsymbol{m} - \boldsymbol{H}\hat{\boldsymbol{s}}^{L}S = \boldsymbol{m} - (\boldsymbol{H}'\boldsymbol{H})^{-1}\boldsymbol{H}'\boldsymbol{m} \tag{3}$$

With knowledge of the noise variance σ^2, the presence of bad data can be readily detected by the χ^2-test [6, Chapter 7], which declares bad data to be present when $\|\boldsymbol{r}\|_2^2/\sigma^2 > \chi^2_{1-\alpha,2q}$, where $\chi^2_{1-\alpha,2q}$ is the tail value of $1-\alpha$ for χ^2-distribution with $2q$ degrees of freedom and α is the detection confidence level.

Once the presence of bad data is detected, the system will proceed with the bad-data processing. There are two ways to deal with bad data. One can either start by determining the locations of the bad data, or by estimating the bad-data locations and values simultaneously.

III. Bad-Data Processing by First Estimating the Bad-Data Positions

In this section, we will introduce two algorithms that first estimate the position and then further deal with the bad data based on the estimated position information. Before the algorithm development, it would be interesting to discuss three strategies to utilize the bad-data position information, namely joint estimate, estimate and subtraction, and simply removal.

In the joint estimate strategy, the bad-data values at those positions $\boldsymbol{b}_k$ are added into the original unknown state $\boldsymbol{s}$ to formulate a new state with more unknowns. Then, the position information of the bad data is utilized to formulate a linear system with larger dimension. Accordingly, the state estimate $\hat{\boldsymbol{s}}$ can be extracted.

In the estimate-and-subtraction strategy, the position information is further utilized to estimate the values of the bad data to obtain $\hat{\boldsymbol{b}}_k$. Then, the bad data will be subtracted from the measurement $\boldsymbol{m}$ to obtain the state estimate $\boldsymbol{s}$.

In the third strategy, since the positions are known, the contaminated measurement will be directly removed from $\boldsymbol{m}$. This will result in a reduced system model.

At a first glance, it seems that the first strategy is the most comprehensive and thus will have the best performance, while the last one is the simplest and expected to provide the worst performance. However, our work [7] shows that all these three strategies will result in the identical state estimator. This means that one can adopt the simplest strategy, i.e. the bad-data removal. In the following, we will introduce two algorithms that locate the bad data and remove it one by one.

A. Largest Residual Removal (LRR)

Traditionally, the measurement with the largest residual is usually considered bad and removed in power grid SE as described in Fig. 3 [6]. Essentially, in this algorithm, it will remove the worst measurement under *a posteriori* analysis one by one. Obviously, this algorithm does not take advantage of the system structure $\boldsymbol{H}$ for its bad-data location.

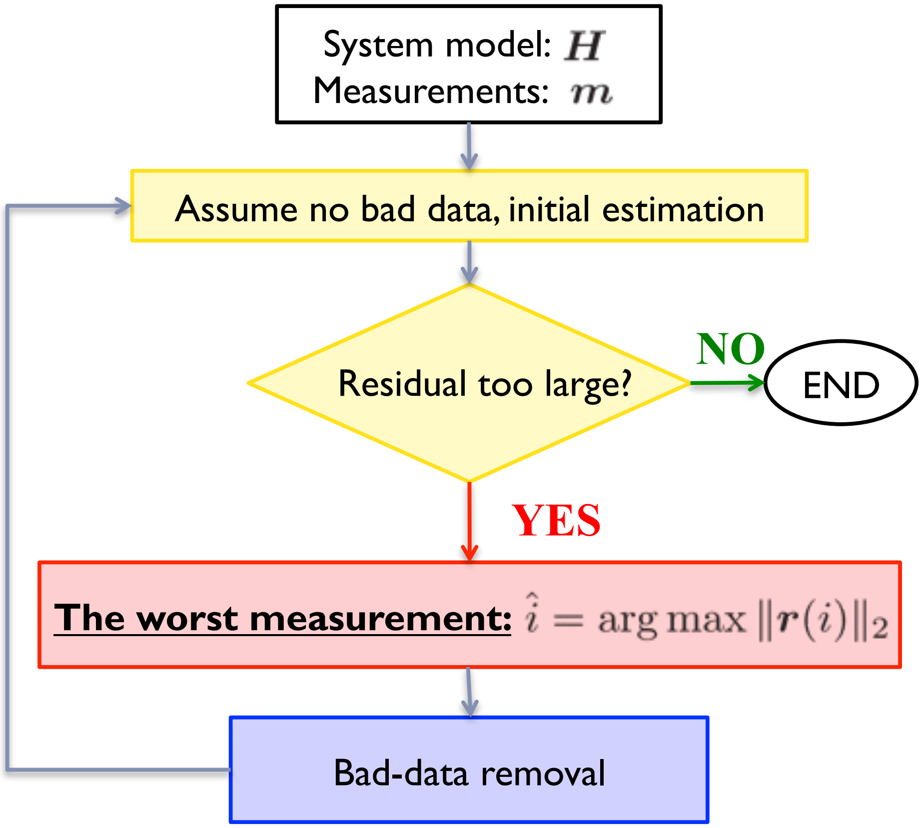

Fig. 2. State estimation with largest residual removal.

B. Projection and Minimization

In order to improve the bad-data removal performance, we propose a new algorithm to better find the bad data locations. Instead of looking at the single measurements, we are looking at the pattern of the resultant residual vector $\boldsymbol{r}$. Each time, we select the most likely position of the bad data whose residual pattern $\boldsymbol{r}_i$ would be closest to the true residual vector with detailed expressions in [7].

IV. Bad-Data Processing by Joint Bad-Data Position and Value Estimation

Accounting for the sparsity of bad data, $\boldsymbol{b}$, the joint estimation of bad data and state can be formulated as a sparsity regularized minimization problem as follows:

$$(\hat{\boldsymbol{s}}, \hat{\boldsymbol{b}}) = \arg\min_{(\boldsymbol{s},\boldsymbol{b})} \left(\|\boldsymbol{m} - \boldsymbol{b} - \boldsymbol{H}\boldsymbol{s}\|_2^2 + \lambda \cdot \mathrm{spar}(\boldsymbol{b})\right) \tag{4}$$

where $\mathrm{spar}(\boldsymbol{b}) = \|\boldsymbol{b}\|_0$, but can be replaced with any other approximating sparsity measures, such as $\|\boldsymbol{b}\|_p$ with $p \leq 1$.

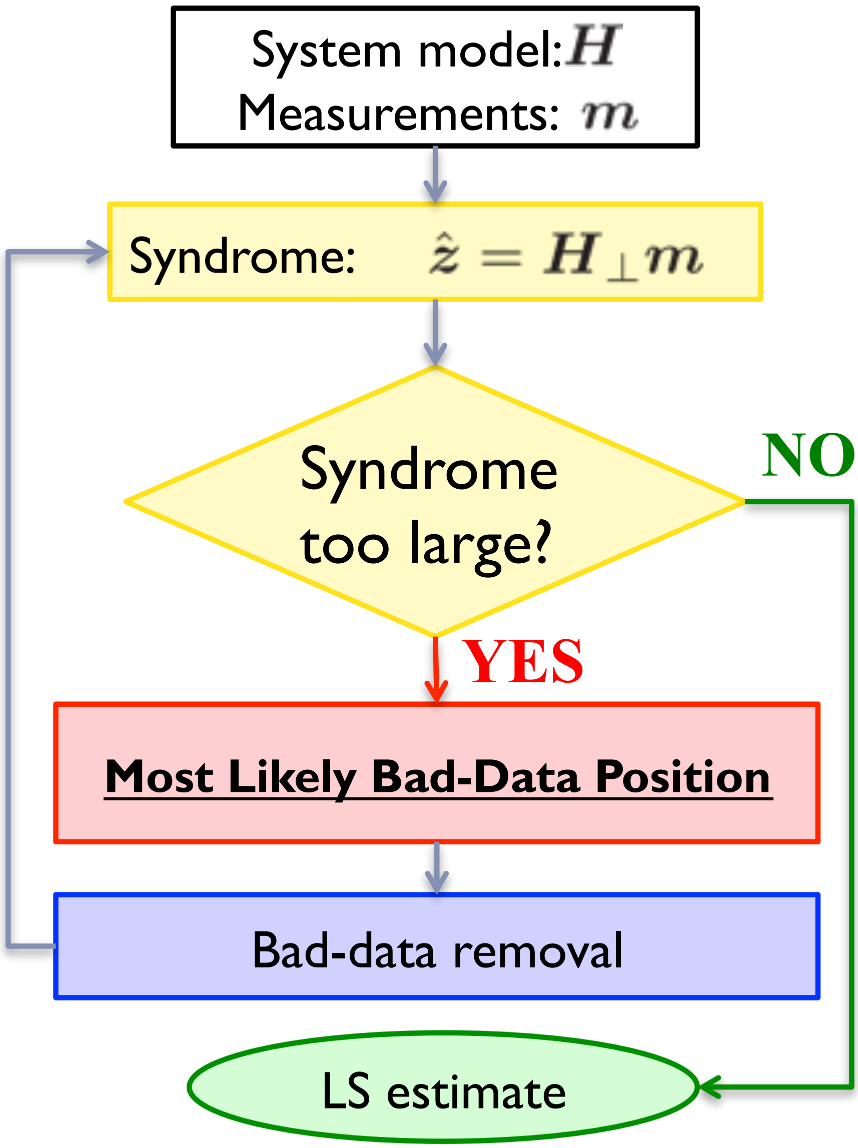

Fig. 3. State estimation with projection and minimization algorithm.

With the conditional estimate $\hat{\boldsymbol{s}}(\boldsymbol{b}) = (\boldsymbol{H}'\boldsymbol{H})^{-1}\boldsymbol{H}'(\boldsymbol{m}-\boldsymbol{b})$, Eq. (4) can be rewritten as:

$$\begin{aligned}\hat{\boldsymbol{b}} &= \arg\min_{\boldsymbol{b}} \left(\|\boldsymbol{m}-\boldsymbol{b}-\boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{-1}\boldsymbol{H}'(\boldsymbol{m}-\boldsymbol{b})\|_2^2 + \lambda\,\mathrm{spar}(\boldsymbol{b})\right) \\ &= \arg\min_{\boldsymbol{b}} \left(\|\boldsymbol{P}_H^{\perp}\boldsymbol{b}-\boldsymbol{r})\|_2^2 + \lambda\,\mathrm{spar}(\boldsymbol{b})\right) \end{aligned} \tag{5}$$

where $\boldsymbol{P}_H^{\perp} = \boldsymbol{I} - \boldsymbol{H}(\boldsymbol{H}'\boldsymbol{H})^{-1}\boldsymbol{H}'$.

Essentially this algorithm try to estimate the bad data as the sparsest solution to the underdetermined equation $\boldsymbol{P}_H^{\perp}$. This is a classic problem called compressive sensing with many algorithms in literature [8].

V. SIMULATIONS

We use the IEEE 14-bus system shown in Fig. 4 for our simulations [14]. The TVE for synchrophasor measurements is 1%, which is the maximum tolerable TVE defined in the standard [1].

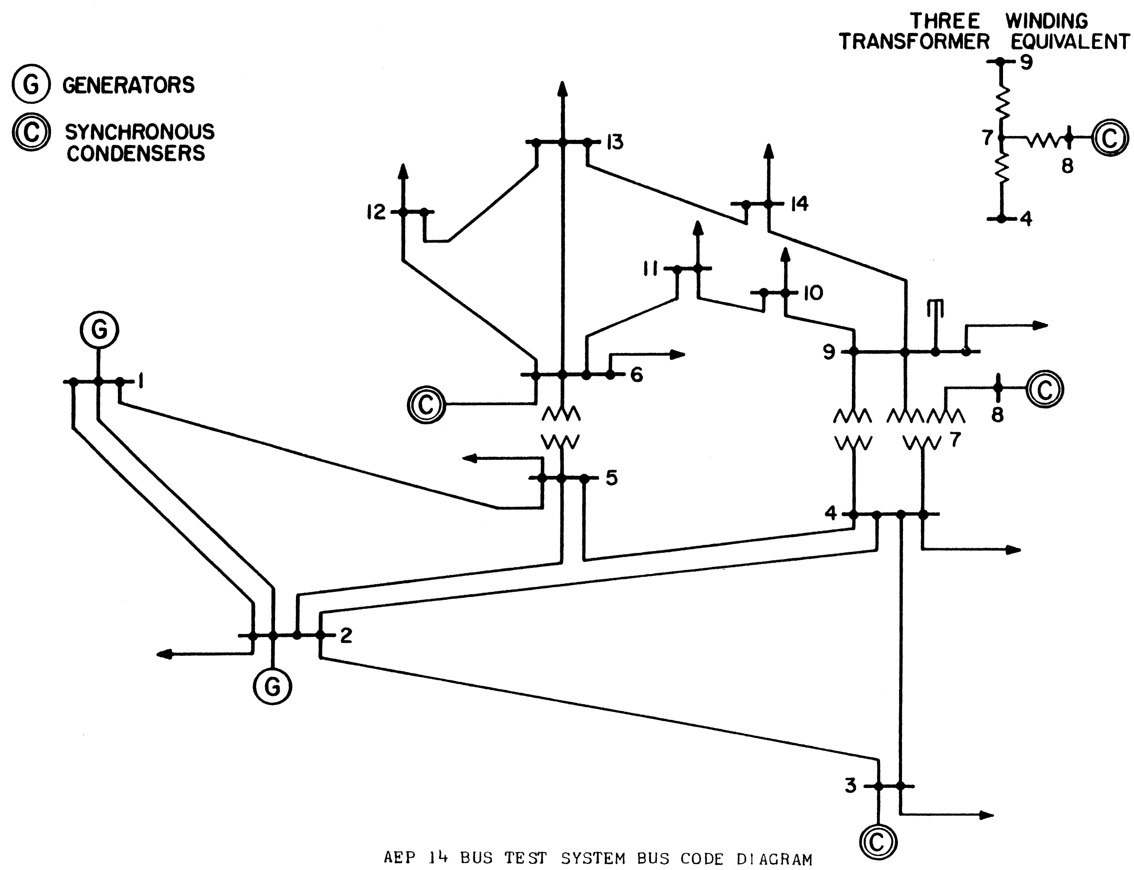

Fig. 4. IEEE 14-bus system.

Of the various sparsity measures and algorithms, here we use l_1-norm with LASSO algorithm [15] and l_0-norm minimization algorithm [16]. Genie-aided solutions with known bad data locations are also included as a reference.

In Figs. 5 and 6, we simulate the case with 14 bus voltage and 14 injection current measurements ($q{=}2p$) and the occurrence of a single bad measurement and 3 bad measurements, respectively. Clearly, the measurements are not redundant enough for LRR to guarantee good performance. In Fig. 5, PM is optimum and perfectly identifies the single bad data as expected. Moreover, none of other algorithms is optimum, even for this single bad data occurrence case. In Fig. 6, with 3 bad data, it can be seen that the performance is ranked as Genie-Aided>PM>l_0-SRM>LASSO-SRM.

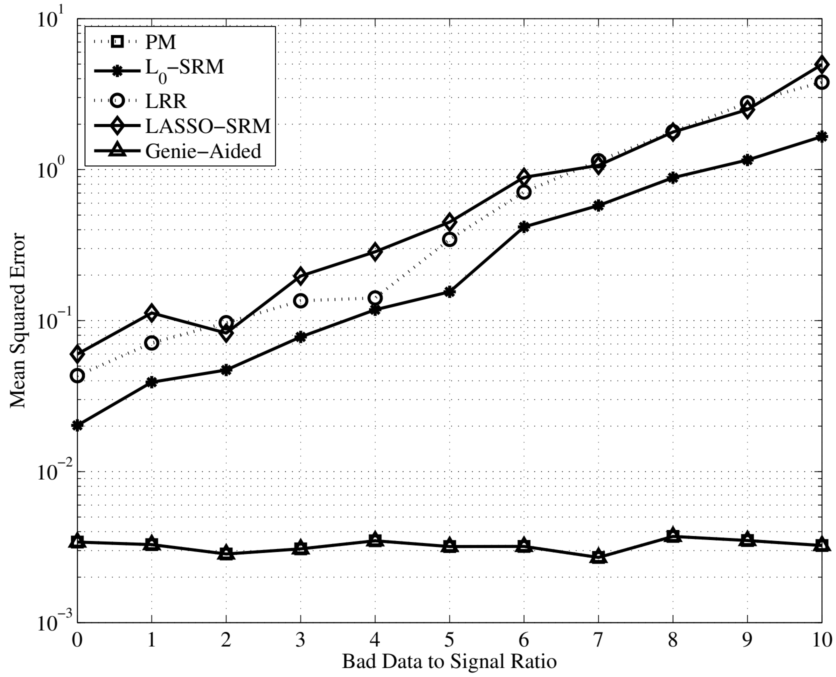

Fig. 5. State estimation performance for measuring the bus voltages and injection currents with 1 bad data.

In Fig. 7, we increase the data redundancy by including all 68 possible measurements, i.e., 14 bus voltage, 14 injection current and 40 line current measurements ($q>4p$). Out of these, 4 bad measurements are randomly generated. The results show that, even with increased bad measurements, performance of all but LASSO is significantly better than those in Figs. 5 and 6. This implies that, in order to improve the estimator performance, it is desirable to include as many measurements as possible. However, LASSO performs worse than in the former cases, which may be caused by the non-fatness of the 54×68 regression matrix $\boldsymbol{H}_{\perp}$. It is worth noticing that, in this case, the measurement redundancy resulted in pretty good performance with LRR.

VI. Conclusions

In this project, power grid state estimation algorithms using PMU measurements with possible bad data are proposed. By explicitly incorporating the bad data into the measurement model, we showed that the conventional LRR does not have performance assurance and developed several more sophisticated algorithms. To deal with the bad data, one can estimate the bad data location and values either separately or jointly. For the former approach, we established the equivalence among joint estimate, estimate and

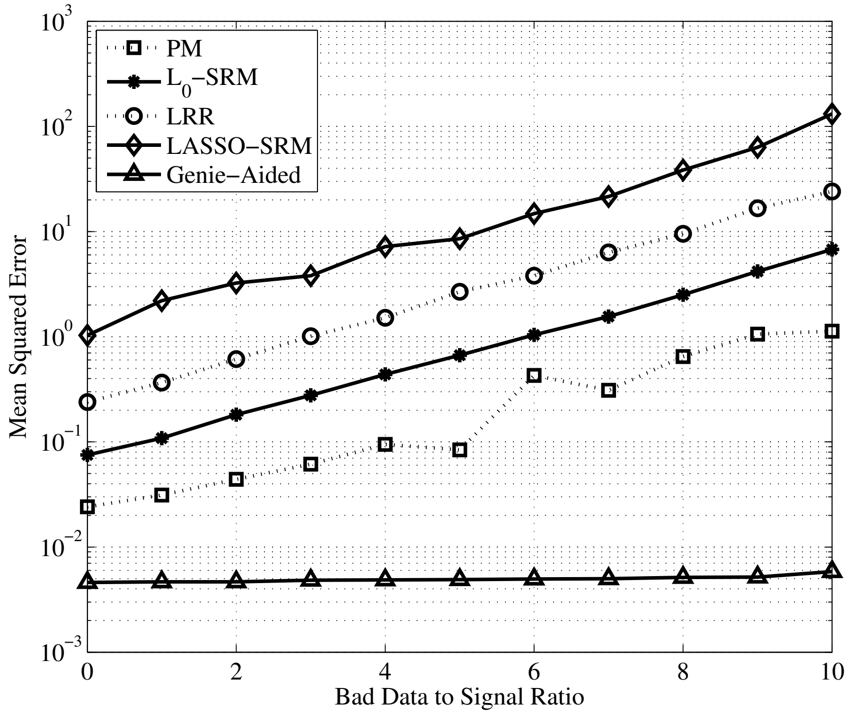

Fig. 6. State estimation performance for measuring the bus voltages and injection currents with 3 bad data.

subtraction, and bad-data removal. Then we developed the projection and minimization (PM) algorithm. For the joint bad data location and value estimation case we formulated a sparsity regularization minimization (SRM) problem and transformed it into a compressive sensing problem. Simulations on the IEEE 14-bus test system with different levels of measurement redundancy and bad data occurrence are provided. Results showed that our PM algorithm has not only the lowest complexity but also the best performance.

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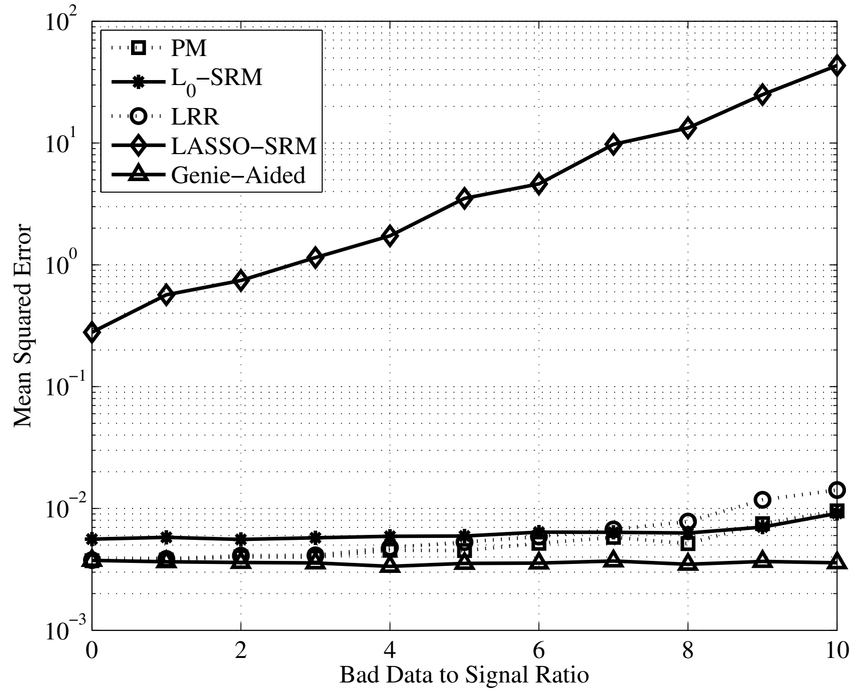

Fig. 7. State estimation performance for measuring the bus voltages, injection currents and all line currents with 4 bad data.

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