The Input Impedances of Slit Antennas
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Although various works on the calculation of the radiation impedances of slit antennas have been carried out, they used only the so-called electromotive force method which has been generally used in the theoretical treatment of ordinary linear antennas. However, the electromotive force method based on the assumption of unknown current distribution is not accurate and has been a past method. A new method, in which the impedance is calculated directly from the ratio of the applied voltage to the driving current by using the current distribution determined by the boundary conditions, has been developed. Therefore, it is better not to use the electromotive force method assuming the magnetic current distributions.

In this paper, we consider general relations between the electromagnetic field due to holes on an infinite perfectly conducting plane and the field due to the dual plates having the same shapes as the holes by introducing magnetic perfect conductor. This consideration is different from that reported by Kotani. Next, by using these relations we calculate directly the input impedance of a slit antenna from the relation of the voltage and the current without using the idea of radiating power used as in the electromotive force method. Finally, general relations between the input impedances of slit antennas and those of planar antennas having the same shapes as the slit antennas are derived. These relations are applied to planer antennas having a constant input impedance.

Outline of the theory is as follows. Let us consider a structure composed of a perfect electric conductor and a perfect magnetic conductor with an electric current source \( J_0 \) and a magnetic current source \( J^*_0 \) in a space with constant permittivity \( \varepsilon \), constant permittivity \( \mu \) and constant conductivity \( \sigma \). Electromagnetic fields in this structure are \( E_1 \) and \( H_1 \). Next we consider another structure having a perfect magnetic conductor and a perfect electric conductor which are replaced with the perfect electric conductor and the magnetic conductor in the first structure, respectively. The electric and magnetic current sources in the second structure are \( -\gamma J^*_0 \) and \( J_0 \), respectively. We found that the electromagnetic fields \( E_2 \) and \( H_2 \) in the second structure are given by

\[
E_2 = -H_1, \quad H_2 = \gamma E_1, \quad (1)
\]

where \( \gamma \) is given by

\[
\gamma = \frac{\varepsilon + \sigma}{\jmath \omega \mu}. \quad (2)
\]
Next we consider symmetric electric current source and antisymmetric magnetic current source about a plane of $x=0$. Under the condition of symmetric geometry, we have proven that the field is electrically symmetric (E-symmetry) and if we insert a plane of perfect magnetic conductor with an arbitrary shape on the plane of $x=0$, electromagnetic field does not change.

When there are symmetric electric current sources with a plane of perfect electric conductor, we can insert a plane of perfect magnetic conductor without changing the electromagnetic fields $E_1$ and $H_1$. When the perfect electric conductor and the perfect magnetic conductor are replaced with the perfect magnetic conductor and the perfect electric conductor, the electromagnetic fields $E_2$ and $H_2$ are again E-symmetry, and we can remove the perfect magnetic conductor without changing the electromagnetic fields. By using eq. (1), we obtain the following equation,

$$E_2 = -H_1,$$
$$H_2 = \gamma E_1,$$
$$E_2 = H_1,$$  \[ x > 0, \]
$$H_2 = -\gamma E_1, \quad x < 0. \quad (3)$$

The slit antenna driven by the electric source shown Fig.1 (A) can be replaced with the slit antenna driven by the magnetic source as shown Fig.1 (B). By using relations given by eq. (3), we have clarified the relation between the slit antenna shown in Fig. 1 (B) and the planar dipole antenna shown in Fig. 3 (C), and we obtain a relation between the driving point impedance matrix $[Z]$ of the slit antenna and the driving point impedance matrix $[Z']$ of the planar dipole, which is dual to the slit antenna, has been obtained using line integrals as

$$[Z] = [Z']^{-1}/4\gamma = (60\pi)^2[Z']^{-1} \quad \text{(in vacuum space)} \quad (4)$$

The above equation has been previously derived by assuming magnetic current distribution and using the idea of radiating power. However, we obtain eq.(4) directly from the relation between the driving current and the voltage without the assumption of magnetic current distribution. According to the present theory, it is noted that eq. (3) can be used for antennas having arbitrary shapes.

![Fig. 1 Relation between slit antenna and planar dipole antenna.](image-url)
By using this theory, input impedance of a slit antenna with a total length of $2l$ can be obtained by using Hallen’s equation as

$$Z = j60\pi^2 \frac{\sin kl + \frac{\beta_1}{\Omega} + \frac{\beta_2}{\Omega^2}}{\Omega \cos kl + \alpha_1 + \frac{\alpha_1}{\Omega}}. \quad (5)$$

Moreover, it is noted that the input impedance at the terminals $a$ and $b$ of rotationally symmetric and line symmetric planar structures shown in Fig. 2 is $Z = 60\pi \Omega$ which does not depend on the frequency and their shapes.

Acknowledgments
The author is thankful to professor Uda and associate professor Sunouchi at Tohoku University for their helpful advice.

Fig.2 Planar structures whose impedances at terminals $a$ and $b$ are the same.