Below is an excerpt from:
Babylonian Mathematical Texts I. Reciprocals of Regular Sexagesimal Numbers
Author: A. J. Sachs; Source: Journal of Cuneiform Studies, Vol. 1, No. 3 (1947), pp. 219-240
Published by: The American Schools of Oriental Research
Stable URL: http://www.jstor.org/stable/1359434; Accessed: 06-08-2015 15:56 UTC

Journal of Cuneiform Studies, Vol. 1
5) $[33,20]$
6) $[1,6,40]$
7) $[2,13,20]$
12) $1,11,6,40$
13) $2,22,13,20$
14) $4,44,26,40$
19) $2,31,42,13,20$
20) $5,3,24,26,40$
§23. The first number on the reverse should be $10,6,48,53,20$, but instead of $10, \ldots$ the tablet has 10 followed by a vertical wedge from which a diagonal is suspended. Using the abbreviation vpd for this combination of vertical-plus-diagonal, we present the following list to show what the reverse reads and what we should expect:

| Line | Text | Calculation |
| :---: | :---: | :---: |
| 21 | $10 \mathrm{vpd}, 6,48,53,20$ | 10,6, 48, 53, 20 |
| 22 | $20 \mathrm{vpd}, 13,37], 46,40$ | 20, 13, 37, 46, 40 |
| 23 | 40 vpd, $27,15,33,20$ | 40, 27, 15, 33, 20 |
| 24 | 1, $20 \mathrm{vpd}, 54,31,6,4,40$ | 1, $20,54,31,6,40$ |
| 25 | 2, $40 \mathrm{vpd}, 1,49,2,13,20$ | 2, 41, 49, 2, 13, 20 |
| 26 |  | 5, 23, 38, 4, 26, 40 |
| 27 | 10,40 vpd, 7, 16, 8, 53, 20 | 10, 47, 16, 8, 53, 20 |
| 28 | 21, [20 vpd, 14, 32, 1] , 46, 40 | 21, 34, 32, 17, 46, 40 |
| 29 | $42,40 \mathrm{vpd}, 29,4,35,33,20$ | 43, 9, 4, 35, 33, 20 |
| $30^{12}$ | 1, 25, $20 \mathrm{vpd}, 58,9,11,6,40$ | 1, 26, 18, $9,11,6,40$ |
| 31 | 2, 50, 40, 1, 56, 18, 2\%, 13: 20 | 2, 52, 36, 18, 22, 13, 20 |
| 32 | 5, 41, 20, 3, 52, 36, 44, 26, 40 | 5, 45, 12, 36, 44, 26, 40 |
| 33 | $[11,7$ R, 40, 7, 45, 13, 28, 53, 20 | 11, $30,25,13,28,53,20$ |
| 84 | [22, 44, 20, 15, 30, 26, 57, 46, [40] | $23,0,50,2 \%, 5 \prime 7,46,40$ |
| 35 | [45, 30, 40, 31, 0, 53, [E5, 33, 20] | $46,1,40,58,55,33,20$ |
| 36 | $[1,31,1,20,1], 2,1,\left[⿶^{\prime} 7,51,6,40\right]$ | $1,32,3,21,47,51,6,40$ |

Let's recreate this doubling calculation sequence from line 19 through line 36 on a three table base 60 Salamis Tablet.
19) $2,31,42,13,20$ at right.

Doubled below.
[ $2 \Rightarrow 5-1,1 / 2 \Rightarrow 1]$


Doubled below. [10+10 $\Rightarrow 30+10$ ]



Below, make readable: 20) 5,3,24,26,40.


Below, top digit of right table moved to bottom digit of middle table. Splitting the number into two parts here implies the use of an abacus with 5 digits per table and at least two tables.

Below, made readable: 21) $10 \mathrm{vpd}, 6,48,53,20$ [50 $=60-10 \mathrm{w} / 2$ tokens instead of 3]


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Right, double.
Below, make readable: 22) $20 \mathrm{vpd}, 13,37,46,40$


Below, double.

Below, make readable: 23) $40 \mathrm{vpd}, 27,15,33,20$


Below, double.

Below, make readable:
24) $1,20 \mathrm{vpd}, 54,31,6,40$


At right, double.
Below, make readable:
25) 2, $40 \mathrm{vpd}, 1,49,2,13,20$


Below, make readable:
26) 5, $20 \mathrm{vpd}, 3,38,4,26,40$


Below, make readable:
27) $10,40 \mathrm{vpd}, 7,16,8,53,20$



Next the top right digit should be accumulated into the middle bottom digit, but is not. Instead, double both parts; below.


Below, double.


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At right, doubled.
Below, start make readable by adding zero pairs.


Below, a next step in make readable.


Below, doubled.



Below, a next step in make readable.


Below, final step in make readable:
28) $21,20 \mathrm{vpd}, 14,32,17,46,40$


At right, make readable:
29) $42,40 \mathrm{vpd}, 29,4,35,33,20$

Below, simplify before doubling.


Below, doubled.


Below, moved middle table tokens to left table, then unit


Below, made readable:
31) $2,50,40$ (vpd), $1,56,18,22,13,20$


Below, doubled and made readable
30) $1,25,20 \mathrm{vpd}, 58,9,11,6,40$


This is
necessary to continue the calculations and may be verification that a three table abacus was used.

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At right, simplify before doubling.
Below, doubled.


Below, made readable:
32) 5, 41, 20 (vpd),3, 52, 36, 44, 26, 40

Below, doubled.


Below, made readable:
11, 22, 40 (vpd), 7, 44, (vpd), 1, 13, 28, 54, 20.
But here the scribe transfers the top digit of the right

table to
the
bottom digit of the middle table, (at left), so he knows how to do
 this! ...

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... Result (at right):
33) 11, 22, 40 (vpd), 7, 45 (vpd), 13, 28, 54, 20

Below, simplify prior to doubling.


Below, made readable:
34) $22,45,20$ (vpd), 15, 30 (vpd), 26, 57, 46, 40


Below, doubled.


Below, doubled.



Below, doubled.


Below, transfer top digit of right table to bottom digit of middle table and read:
36) $1,31,1,20(\mathrm{vpd}), 1,2,1$ (vpd), 47, 51, 6, 40


At this point Sachs writes, "The remainder of the reverse is broken away; there would have been enough room for about four lines of writing, assuming that our restoration of the obverse is correct." Four more doublings would still leave the top digits on all three tables empty, so the three table abacus would have taken care of it.

Now let's combine the left and middle tables to see if we get Sachs "Calculation" result.

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Right, move middle table tokens to left table.
Below, add and read:
1, 32, 3, 21, 47, 51, 6, 40


Which agrees with Sachs.

