Inventing the 'black box': mathematics as a neglected enabling technology in the history of communications engineering

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Historians of telecommunications have tended to concentrate on enabling technologies such as the vacuum tube, the transistor, the microprocessor, and so on – or on the socio-political aspects of developing, regulating, and managing large-scale communication systems. Yet just as significant historically – but largely neglected by historians of technology – is an approach to modelling, analysis and design based on a quintessentially 'communications engineering' use of mathematics. This approach, ultimately characterised by terms such as 'linear systems theory' and 'black box analysis' is still a key factor in the development of communications devices and systems. In this paper I shall look at some aspects of the history of mathematical modelling in communications engineering. Rather than try to tell any coherent story or stories I shall pick out a number of crucial features of such modelling, and I shall also attempt, albeit very briefly, to set this in the wider context of telecommunications technology, including instrumentation and simulation.

Let me begin with two quotations. The first is from W. A. Atherton's general history of electrical and electronic engineering entitled *From Compass to Computer*. He writes:

> The mathematical treatment of circuits has passed through several stages of development. Originally mathematical statements were made describing the properties of individual components and groups of these components, which was sufficient for much 19th century work on telegraphs and telephone networks. Later researchers analysed more complex circuit structures by breaking them down into smaller simpler sections for which mathematical statements could be made; these sections were then used to analyze the whole complex structure. This stage began around 1900 [… and] out of this work emerged such disciplines as line theory, circuit theory and network analysis. Perhaps the final stage, as exemplified by network synthesis, is the ability to synthesize a circuit by mathematically stating the function it is to have and then evolving the design that will achieve it. (Atherton, p. 226)

The second quotation is from a far more technical paper on the history of circuit theory, written by V. Belevitch, himself an important contributor to the discipline, to commemorate the fiftieth anniversary in 1962 of the IRE, the (American) Institute of Radio Engineers. In Belevitch's view:

> Long before 1914 circuit theory had emerged, from general electromagnetic theory, as an independent discipline with original concepts and methods. The conception of a circuit as a system of idealised lumped elements is already firmly established – drawings of Leyden jars and rheostats have gradually disappeared in favour of the now familiar graphical symbols. This assumes, at least implicitly, that a resistor is considered as a 2-terminal black box defined by the relation $v = \mathrm{R}i$, rather than as a physical device made of metal or carbon. (Belevitch, p. 848)

Now, while both these quotations contain a good deal of truth, they also prompt us to exercise a little caution. The first presents us with a rather unproblematic view of the development of the use of mathematics in communications engineering – a somewhat 'Whig history' to which, I suspect, most practising communications engineers would subscribe. And the second (the phrase 'at least implicitly' is a bit of a giveaway) should sensitise us to the temptation to ascribe to the past the viewpoint and assumptions of the present. In this paper I shall try to tease out some of the historical context, and some of the processes that were involved in the broad historical development outlined by Atherton.

At the risk of jumping ahead, let me lay out my general argument – essentially, that:

- from the end of the nineteenth century, communications engineers developed a new approach to the use of mathematics, initially through the use of phasors and then through increasingly sophisticated time- and frequency-domain modelling;
- the abstraction of system components led to a 'meta-language' in which the manipulation of circuit configurations and other symbolic representations became a natural consequence of – and even alternative to – the mathematics;
- this approach encouraged engineers to concentrate on the input-output behaviour of devices and networks, thus leading to 'black box' and 'systems' ideas;
- systems ideas that originated with communications engineering were extended to other domains, such as control engineering, although the process of dissemination was complex;
- a highly significant synergetic relationship developed between the mathematical modelling of systems and components, the design of instrumentation, and the design of devices and systems;
- the key concepts of orthogonality and duality were crucial in the forging of the links between theories, models, instruments and designs

Phasors

Perhaps a suitable point to start is with the application of complex numbers to electrical circuits, something that was promoted by a number of important figures in the last decade of the nineteenth century (including A. E. Kennelly and O. Heaviside). The idea was elaborated and popularised above all by Charles Proteus Steinmetz. In 1893 the latter gave a lengthy address to the International Electrical Congress in Chicago. In this talk he clearly stated the advantages of the 'vector' [phasor] approach:

> The method of calculation is considerably simplified. Whereas before we had to deal with periodic functions of an independent variable 'time', now we obtain a solution through the simple addition, subtraction, etc of constant numbers … Neither are we restricted to sine waves, since we can construct a general periodic function out of its sine wave components … With the aid of Ohm's Law in its complex form any circuit or network of circuits can be analysed in the same way, and just as easily, as for direct current, provided only that all the variables are allowed to take on complex values. (Steinmetz, 1893)

(Incidentally, of this paper, Steinmetz later wrote: " ... there was no money to publish the Congress paper, and the paper remained unpublished for years, and the symbolic method unknown" (*IEEE Power Engineering Review*, 1996). Yet the paper did appear in German the same year as the Congress, and it is from this source that the above citation is taken.)

Although the use of complex numbers appears to have mystified many early electrical and electronics engineers – Steinmetz's Chicago talk is stated to have been met with much incomprehension, for example – the adoption of the phasor approach was highly significant for future developments in two particular ways. First, as noted by Steinmetz himself in the previous quotation, it allowed quite complicated calculations in the time domain to be replaced by much simpler ones in terms of frequency. Second, it was indeed an important step towards the 'black box' concept. The defining equations for resistors, capacitors and inductors were all subsumed into a generalised, complex version of Ohm's relationship; and even if it would be premature to talk of 'implicit 2-terminal black boxes' at this time, such a representation of components as complex impedances was clearly a great conceptual step. Furthermore, increasing attention in the manufacture of such devices was consequently paid to improving the closeness of the approximation of the component behaviour to the ideal mathematical model.

In the two decades following the first publications on the subject by Steinmetz and Kennelly, phasor analysis became a basic tool of all electronics and communications engineers.

Filter design

During the first two decades of the twentieth century communications engineers made enormous progress in the use of time- and frequency-domain techniques, and Heaviside's operational calculus was put on a much firmer mathematical footing by G. A. Campbell, J. R. Carson, and others. The key to the effective exploitation of bandwidth for both carrier telegraphy and the newer telephony was frequency-division multiplexing (FDM). And for successful FDM, wave filters with quite stringent pass-band characteristics were needed in order to select the desired channel without excessive distortion.

The wave filter was invented independently by Campbell in the USA in 1909 and K. W. Wagner in Germany a few years later. The claim I wish to make here is that in developing design techniques for such filters, communications engineers started to use mathematics in a radical new way. In particular, circuit diagrams became what we might term a 'meta-language' for the mathematics. Consider, for example, the set of diagrams shown in Figure 1[*], taken from a paper by Campbell (1922), the first of a series of papers on wave filters that appeared in the *Bell System Technical Journal* over the next few years. Here we see a fairly complex filter first of all represented as a series of interconnected idealised blocks, then elaborated as a set of equivalent circuit configurations. What I believe is beginning to emerge at this time is the distancing of circuit diagrams in the design phase from their eventual implementation: the manipulation of circuit *component* symbols (in general, complex impedences) becomes an attractive alternative to the manipulation of *mathematical* symbols. Ultimately this will lead to the use of prototype circuits in filter design that bear little or no resemblance to the ultimate electronic circuit.

Figures 2 - 4, which are taken from 1923 and 1924 papers by O. J. Zobel illustrate even more clearly this trend. Note in particular his use of the term 'general linear transducer', in which a linear network is completely defined by its transfer function and terminal / image impedances, shown in Figure 2. This is one of the earliest appearances in the technical literature of the mature 'black box' approach (although Breisig had used similar ideas in 1921 in his characterisation of a two-port network by its voltage equations). Figure 3 shows how circuit diagrams and mathematical expressions were becoming equivalent; while Figure 4 illustrates some of the transformations and manipulations that filter designers began to carry out using what I have called the 'meta-language' of circuit symbols.

Now, I do not wish to give the impression that this approach exploded fully-formed on to the filter design scene in the early 1920s! The work of Campbell, Zobel and others drew on a great deal of earlier work. For example, as early as 1899 Kennelly had established the star-delta transformation (exemplified in Figure 1 above); and the concept of duality (the recognition of the fundamental relationship between such pairs as current-voltage, parallel-series, mesh-node, etc) was also known by the early years of the century. Nevertheless, it does seem that by the early 1920s wave filter design was already pointing firmly towards the 'black box' and 'linear systems' approach that was to become so crucial over the following decades.

[*] Unfortunately it is not possible to include the figures from the *Bell System Technical Journal* in this draft. Negotiation is in progress with AT&T, the copyright holders, and it is hoped that a revised draft can be made available in due course.

The ‘mathematicization’ of filter design – and the corresponding further development of the electrical ‘meta-language’ – continued apace during the 1920s. Foster (1924) and Cauer (1926) are two classic papers. As a later commentator (Zverov, 1966) put it:

> Foster partitioned the given rational function into a sum of partial fractions that could be identified easily as a series connection of impedances or a parallel connection of admittances [duality again]. Wilhelm Cauer expanded the rational function into a continued fraction representing a ladder network. Each method gave two alternative networks, which were called canonical forms because thay could always be obtained from a realizable immittance function and because they employed a minimum number of elements.

What we see here, then, is the direct translation of quite sophisticated mathematics (partial fractions and continued fractions) into circuit elements, in a way which allowed the circuit designer to move at will between the two representations. The fact that this subsequently became common currency in electronics design should not blind us to the novelty of the approach.

This approach went much further in subsequent decades, with considerable blurring between real circuit components and mathematical abstractions. Writing in an editorial in the September 1955 edition of the *IRE Transactions on Circuit Theory*, the editor, W. H. Huggins, argued that:

> ... modern circuit theory is concerned but little with the circuit as a physical entity and, instead, has become increasingly involved with ... signals ... Thus the usual circuit diagram may be regarded as a pictorial form of a signal flow graph [ie an alternate mathematical representation]

As the years went by, electronics engineers introduced a whole set of new, ideal, circuit elements – such as the gyrator, nullator, norator, or supercapacitor – which became just as ‘real’ to the filter designers as resistors, capacitors and operational amplifiers. My colleague Chris Dillon remarked on reading a draft of this paper:

> It is certainly the case that all these things are ‘real’ in the sense that they lead to useful ways of talking about the constraints within a network without writing down mathematics *per se*. There is no doubt that this particular ‘pictorial’ representation of circuit constraints is very powerful and – perhaps – is as formal as a mathematical description in that it is no more and no less than an unwrapped version of the mathematics (with the proviso that Kirchhoff’s laws are assumed valid to provide the binding constraints at the interconnections of the ‘components’, rather like versions of the ‘equals’ sign). It is a way of writing the mathematics that engineers have learnt to see as connections of components – and to be able to go backwards and forwards between these domains effectively doing very complex mathematics by modifying the circuit diagram or using network transformations (which are just identities in the mathematical sense), and often having a very good idea what the result of changes will be.

Systems thinking

A second major argument of this paper is that the approach to mathematics developed initially by communications engineers led to a much wider applicability in technological analysis, modelling and design. The work presented so far, however, does seem to have been directed only to the narrower domain of electronics design, particularly for telegraphy and telephony. But some engineers in the 1920s were already making the intellectual leap beyond their own discipline. The German Karl Küpfmüller, for example, made the point as early as 1928:

> Practical problems are generally concerned with the relationship between two system variables; for example, between a ‘force’ S_1 and a dependent variable S_2. Assuming a linear system, then in the steady state

> $S_2 = U.S_1$
>
> [...] where U is a factor independent of S_1. Using symbolic vector notation, U is in general complex, and depends on frequency:
>
> $U = A(\omega) \exp[-ia\omega)]$
>
> We shall call A(ω) the transmission factor [gain] and a(ω) the transmission angle [phase shift] of the system.
>
> The variables S_1 and S_2 can be completely arbitrary in nature; S_1 for example could represent a mechanical force, an electrical voltage, or a sound pressure level; S_2 a velocity, a magnetic field, or the strength of an electric current. [...] The complex transmission factor provides a complete description of the system with respect to the variables S_1 and S_2 [and ...] determines the form of the transient behaviour. (Küpfmüller, 1928, p.19-20)

This appears to be the first explicit suggestion that the approach developed for electrical systems could also be applied to other technological devices. What is more, Küpfmüller had also been looking beyond the confines of deterministic models. In a 1924 paper he analysed filters characterised by the 'ideal' characteristic of constant amplitude ratio and linear phase. Such a filter characteristic is unrealisable and non-deterministic, yet can be used to derive extremely useful practical 'rules of thumb' for system design. For example, Küpfmüller showed that his ideal frequency domain constraints could be translated into corresponding time-domain constraints on rise time and transient response. In other words, the precise way in which the filter was implemented was, to some extent immaterial: any bandlimiting brought with it an inescapable restriction on time-domain behaviour. It's another example of duality: the stricter the bandlimiting, the greater the rise time – something that was generalised as the notion of *time-bandwidth product*.

Küpfmüller's approach was not immediately accepted by the profession, as he explained fifty years later:

> There were serious objections to my 1924 paper on transient response in wave filters. At that time it was usual to calculate transient response only for concrete devices such as cables, transmission lines, networks, etc. That it was possible to make abstract assumptions about the characteristics of systems, and thereby, particularly in complicated cases, arrive simply at approximate conclusions about transient response, led to many objections being raised. Moreover, neither did the phrase 'systems theory' for such an approach find immediate acceptance. (Küpfmüller, 1977, p.775)

By the mid 1930s, however, general linear systems theory (at least for deterministic signals) was firmly established within communications engineering, and the first textbooks to deal with the topic had appeared. One classic is *Communication Networks* published in 1935 by Ernst Guillemin, who subsequently proved to be a prolific writer of highly influential student texts (and a great engineering educator at MIT). In the preface to the 1935 volume he pointed out the wider applicability of the subject matter:

> The communications viewpoint is chosen for the more detailed discussions because it affords at present a more suitable vehicle for illustrating the fundamental principles. […] Nevertheless these ideas and principles are too general in nature to remain confined to one field of application, and are more recently beginning to make their appearance in the treatment also of power problems.

Guillemin was well-acquainted with the German literature on filter design, having studied in Munich under Arnold Sommerfeld and he incorporated Cauer's approach to filter synthesis as well as

Küpfmüller’s general linear theory into this volume. A new type of language is also reflected in the book, in which the ‘mathematics’ starts to become remarkably concrete. For example, when discussing Foster’s reactance theorem, Guillemin writes:

> a driving-point reactance function is uniquely specified by the location of its internal zeros and poles plus one additional piece of information which […] takes the form of a multiplying factor …

In this new language the poles and zeros of the filter become just as real as the resistors, capacitors and inductors from which it is constructed. I shall return to this point later.

Feedback systems and control engineering

One of the most influential applications of the ‘communications engineering’ way of doing mathematics was to feedback systems: first to the feedback electronic amplifier, and later to other feedback control systems. The classic papers in English are Nyquist (1932) and Black (1934). The story – or, at least, the myth of origin – is too well known to be rehearsed here once more; readers are referred to Bennett (1993) and Mindell (2000) for a discussion and critique. But I want to make three points of particular relevance to this paper:

1. By 1927, when Black conceived the negative feedback amplifier, the notation of electrical / electronic ‘black boxes’ was well established: Figure 5 shows the version of his design given in his 1934 paper, in which the classic abstractions are clear.

2. By the time Black and Nyquist analysed electronic feedback circuits in the frequency domain, their German contemporary Küpfmüller (1928) had already carried out related work in the time domain (prompted by the analysis of an automatic gain control circuit, but developed generically) – including the statement of a valid stability criterion. Figure 6 shows his generic loop – opened, as in the work of Nyquist a few years later – in order to analyse loop stability.

3. The adoption, by engineers concerned with control problems, of the analytical and design techniques of communications engineering and filter design was by no means straightforward and unproblematic.

General systems thinking appears to have emerged independently in the USA, UK and Germany in the late 1930s and during the war. The US context and the role of interdisciplinary activities at MIT and Bell Labs, for example, has been documented reasonably well, so I shall restrict myself here to three quotations from British and German engineers I interviewed in the early 1990s. All three were pioneers of control engineering, a discipline which benefited enormously from the injection of the communications engineering approach in its development.

First, Arnold Tustin (a British electrical engineer by training), who worked on the famous ‘fire control’ problem:

> I think my personal realisation [of the systems approach] came as I worked with control systems which consisted of a number of operations in sequence - the fire control problem was classic, of course, with target detection, tracking, prediction, and gun laying. As we worked on such systems, combining the responses of the various elements to get the total response, it became natural to think in general system terms, applying identical methods even though one element might be electrical, another mechanical, hydraulic or even human.

Second, Winfried Oppelt, a German engineer (initially an applied physics graduate) who worked primarily on flight control systems:

> Systems thinking emerged very slowly, and really in a quite anonymous way - no single person is responsible, even though some individuals, such as Küpfmüller, were more alive to the concept at an early stage than others. We also devoted a lot of effort to this aspect in the VDI [German engineers' professional body] Specialist Committee. But the development of systems thinking came about very gradually, as part of group work and discussion. It was a long time before the general applicability of control concepts was understood. For example, F. Strecker, at Siemens, had come up with a version of the Nyquist criterion in 1930, although it wasn't published at that time, and most people, including myself, were not aware of the work at all until much later. After the war he incorporated his original paper into a book, but if you read it you can see that from the general control point of view he had not taken the decisive step, at least not in my opinion. The shift from a 'communications way of thinking' to a much more general 'control way of thinking' was the fundamental step, and that had still to be made.

And finally, Hans Sartorius, a German engineer who was concerned primarily with process control problems at Siemens (although, crucially, he had been trained in *Elektrotechnik*):

> When I came to control engineering [late 1930s], through pure chance, there was a great variety of different approaches to theory and the use of mathematics – every discipline went its own way, different approaches for torpedo control, for thermal installations, etc. You couldn't really talk of a systems approach. But we recognised that it was really all the same thing, the same language, you could use the same symbols. In this way systems thinking emerged automatically. […] That would have been just before the war, I think, or in the early days of the war.

These three engineers report a typically complex and 'messy' background to what is often portrayed as a straightforward technological development – in this case the application of communications ideas, such those of Nyquist, Bode and others, to the control context. Although none of the interviewees were communications engineers, they had all been exposed to the mathematical approaches discussed earlier. And both Oppelt and Sartorius wrote books in the 1940s which applied the 'communications way of thinking' to a more general context, including the dissemination of Küpfmüller's work. Yet all three testified to the rather slow and difficult process in adopting this 'systems' way of thinking, and the novelty of applying general systems ideas outside their 'natural' home of communications and electronics engineering. The detailed history of this seminal shift in thinking is still not clear.

Models and 'reality'

In the 1920s and 1930s the modelling approach developed by communications engineers led to considerable unease about quite how the mathematics related to the 'real world'. For some, the frequency-domain models were particularly problematic. John Bray notes that:

> It seems remarkable now that in the 1920s there were some, including the eminent scientist Sir Ambrose Fleming, who doubted the objective existence of the sidebands of a modulated carrier wave, regarding them as a convenient mathematical fiction … (Bray, p. 71-2)[*]

And in the conclusion to his book he returns to this in a personal reminiscence:

> In 1935 […] I entered the Open Competition for a post as Assistant Engineer in the Post Office engineering department. This included an interview with the Chief Engineer, Sir Archibald Gill – an interview during which he asked: "Do you believe in the objective existence of sidebands?" It may seem remarkable that such a question should even have been

[*] A debate took place in the pages of *Nature* on this very issue during the first few months of 1930, sparked off by a short article by Fleming.

> posed , but at the time there was still a lingering controversy […] as to whether sidebands were just a convenient mathematical fiction or whether they were 'real'. Luckily I had witnessed […] a convincing demonstration involving a frequency-swept tuned circuit and a sine-wave modulated carrier, the response being displayed on a primitive electromechanical Dudell oscilloscope and revealed as a triple-peaked curve. So my response to the question was a triumphant "Yes, I have seen them!" (Bray, p. 356)

This is an extraordinarily revealing anecdote. To Bray in 1935, the spectrum of the modulated signal displayed on the oscilloscope was just as 'real' as the corresponding time-varying waveform. In fact, both types of display are fairly remote from 'reality': like a spectrum, an electrical voltage or current can only be revealed by an instrument designed to detect it. The fact that the time-domain representation of such variables seemed (and still seems) so much more natural has more to do with three centuries of natural science and its particular models and conventions than to any ontological distinction[*]. Communications engineers developed their instruments and practical techniques hand in hand with their mathematical models and other symbolic representations. In a sense, of course, this was not so different from the experience of natural scientists; what *was* different, however, was the way the modelling and instrumentation techniques so developed were soon exploited for *design*. So, for example, the frequency-domain models of modulated waveforms led to a whole range of practical techniques which derived directly from the mathematical models: single-sideband, suppressed carrier, originally suggested by Carson in 1915, is one of the most impressive examples of this approach.

Let me give another example, this time from a quarter of a century later. During WW2, linear systems theory made enormous strides, particularly in the handling of random signals and noise (for gun aiming predictors, for example). One particular technique, known as correlation, proved to be extremely fruitful in the theory of signal detection. I will spare you the mathematical details here, but the process of correlation involves taking two waveforms, and comparing their similarity for varying displacements in time against one another. Around 1950, engineers at MIT built an electronic device to carry out this operation and plot the 'correlation function' on a chart recorder. Since then, correlation functions have become just as much a part of the language of communications engineers as spectra, and are implemented in a huge range of modern electronic devices (digital processors proved to be highly amenable to such tasks). So my final claim in this paper is that the interaction and synergy between mathematical models, 'meta-representations' and electronic instrumentation was a decisive aspect of the development of communications engineering.

In their conclusion to their paper reporting the MIT electronic correlator, the authors remarked:

> The method and technique of detecting a periodic wave in random noise presented here may be regarded as a type of filtering in the time domain […] From an engineering point of view, many equivalent operations may be more practicable and feasible in the time domain. For instance, a 'zero' bandwidth filter in the frequency domain corresponds to an extremely stable oscillator in the time domain. At the present time it is much easier to build a stable oscillator than to build a network having zero bandwidth. (Lee et al, 1950)

Yet again we see the modelling flexibility of moving between time- and frequency-domains, and the way in which the mathematics has become something quite different in the hands of the communication engineers. And, most interestingly of all, perhaps, the mathematical 'black box' entitled a 'correlator' has become a real black box that delivers traces on paper of correlation functions.

From the 1950s onwards, the availability of machine computation became increasingly important for the design of communication systems and their components. Zverov points out, for example, that the filter design methods of Cauer and Darlington only became popular with the advent of cheap computation. And with the rise of digital technology, there has been a remarkable convergence of design tools, simulation, and implementation. Modern computer tools now employ the same 'meta-

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[*] Although our 'natural' models of light and sound tend to be in terms of spectral notions (colour, pitch).

languages' as the engineer, and the user can effectively carry out a highly sophisticated modelling or simulation activity without employing very much 'mathematics' *per se*.

Towards a conclusion

In this paper I have tried to tease out some of the particularities of the communications engineering approach to mathematics and modelling, in an attempt to understand how this approach evolved. I quoted above a brief extract from Guillemin's 1935 book on zeros and poles in filter specification. I believe that this shift in language is extraordinarily significant and I'd like to begin to sum up by quoting (at some length, I'm afraid) from a recent article by a colleague and myself which looked at the way engineers, particularly electronics engineers, 'do mathematics':

> Now, the mathematical treatment of poles and zeros within the specialist area of complex analysis is highly abstract and complex indeed. In contrast, though, many of the manipulations carried out using the engineering models are remarkably simple and 'non-mathematical'. Such models have led to engineers talking a language very different from that of conventional mathematics. This has turned out to be a remarkably concrete language. For example, system designers describe certain systems as 'possessing' so many poles; they talk about the need to 'place' poles in certain regions or 'move' them to a more favourable position; and they become highly adept at visualising the pattern of poles and zeros on the complex plane in terms of how they want the system to behave. This is an example of electronics engineers 'doing mathematics'. The important point is that the language they use is *not* the traditional language of mathematics, even if the manipulation of their models may be completely analogous to other manipulations of other, more conventionally 'mathematical' models. Moreover, this linguistic shift is more than just jargon, and more than just a handy way of coping with the mathematics; the shift indicates a way of thinking about systems behaviour in which the features of the models are deeply linked to the systems they are describing. So, in the case of our electronics engineer, the poles effectively cease to be just convenient visualizations of mathematical complex variable theory, and become system features which are just as real as the electronic components from which the system has been built. Explanations of physical behaviour also shift in the same direction; at a systems level accounts of how and why a system behaves as it does are often couched in terms not of physical variables such as voltage and current, but in terms of where the poles of the model lie. In this different ontology the system poles are not simply part of a convenient mathematical model, they are what <u>cause</u> the system to behave as it does, as if they had the tangible existence of the physical components and measurable signals present in a system. From an engineering viewpoint, then, this language is a powerful way not only of representing important aspects of systems behaviour, but also of explaining how that behaviour comes about. (Bissell and Dillon, 2000)

Similar points can be made about other applications of mathematics by communications and electronics engineers – the language of spectra, the language of correlation (as indicated earlier) the language of constellations in signal space in modulation theory, ... For example, communications engineers will talk about 'placing' the points in signal space, about how the 'distance between them' is relevant to performance in the presence of noise, and so on.

I shall conclude with one final quotation. It concerns the orthogonal frequency division modulation used in digital broadcasting.

> The carriers have a common, precisely-chosen frequency spacing. This is the inverse of the duration, called the active symbol period, over which the receiver will examine the signal, performing the equivalent of an 'integrate-and-dump' demodulation. This choice of carrier spacing ensures orthogonality (the 'O' of OFDM) of the carriers – the demodulator for one

> carrier does not 'see' the modulation of the others, so there is no crosstalk between carriers, even though there is no explicit filtering and their spectra overlap.
> [...]
> Fortunately the apparently very complex processes of modulating (and demodulating) thousands of carriers simultaneously are equivalent to Discrete Fourier Transform operations, for which efficient Fast Fourier Transform (FFT) algorithms exist. Thus integrated circuit implementations of OFDM demodulators are feasible for affordable mass-produced receivers. (Stott, 1997)

This highly sophisticated, apparently non-mathematical, language exemplifies the way modern communications engineers exploit mathematics. Note, in particular:

- the concrete (and even anthropomorphic) use of language to summarise highly complex technical and mathematical ideas – overlapping spectra, demodulators which do not ‘see’ alien symbols
- the ‘integrate-and-dump’ shorthand for matched filtering and all its mathematical ramifications
- the ‘systems-level’ link between OFDM demodulation and the DFT
- the conceptual and physical ‘black-boxing’ of the demodulation process

Finally – a repeat of the argument

My argument in this paper is, essentially, that:

- from the end of the last century, communications engineers developed a new approach to the use of mathematics, initially through the use of phasors and then through increasingly sophisticated time- and frequency-domain modelling;
- the abstraction of system components led to a ‘meta-language’ in which the manipulation of circuit configurations became a natural consequence of the mathematics;
- this approach encouraged engineers to concentrate on the input-output behaviour of devices and networks, thus leading to ‘black box’ and ‘systems’ ideas;
- systems ideas that originated with communications engineering were extended to other domains, such as control engineering, but not without difficulty;
- a highly significant synergetic relationship developed between the mathematical modelling of systems and the design of instrumentation to aid design and analysis.
- The key notions of orthogonality and duality allowed engineers to work simultaneously with equivalent models of different types; allowed instruments to be constructed which exploited these models; and ultimately allowed designs to be implemented which became increasingly close to the mathematical idealisations.

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