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Reprints from History of Information Sciences

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E.I. Nečiporuk, "Network Synthesis by Using Linear Transformation of Variables", *Dokladi Akademii Nauk SSSR*, 1958, Tom. 123, No. 4, 610-612.

2007

This publication has been written and edited by
Radomir S. Stanković and Jaakko T. Astola.

Contents

Reprints from History of Computing Science and Signal Processing

Study the history of a scientific discipline is a sign of its maturity. When properly understood and carried out, this study is beyond enumeration of facts or giving credit to particular important researchers. It is rather discovering and tracing the way of thinking that leaded to important discoveries. In this respect, it is interesting and also important to recall publications where for the first time some important concepts, theories, methods, and algorithms have been introduced.

In every branch of science there are some important results published in national or local journals or other publications that have been no broadly distributed for different reasons, due to which they often remain unknown for a wider research community and therefore are rarely referenced. Sometimes, importance of such discoveries is overlooked or underestimated even by the inventors themselves. Such inventions are often re-discovered long after, but their initial sources, if traced, remain almost forgotten, usually no frequently referred, and mostly remain sporadically recalled and mentioned within quite limited circles of experts. This is especially often the case with publications in languages different from the English language which presently dominates the scientific world.

This series of publications is aimed at reprinting and, when appropriate, also translating some less known or almost forgotten, but important publications, where some concepts, methods or algorithms have been discussed for the first time or introduced independently on other related works.

R.S. Stanković, J. T. Astola

Dokladi Akademii nauk SSSR

1958, Tom 123, No. 4

MATEMATIKA

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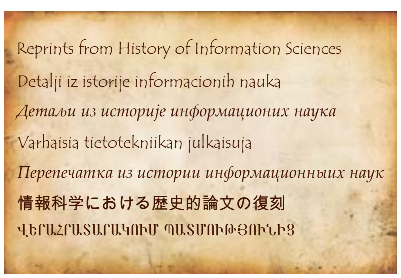

Translated by Radomir S. Stanković

Network Synthesis by Using Linear Transformation of Variables

E.I. Nečiporuk

Abstract

Presented by the academician V.I. Smirnov 3. VII 1968.

1. It is considered the task of synthesis of a network that realizes a Boolean function $f(x_1, x_2, \dots, x_n)$. Denote $x = \{x_1, x_2, \dots, x_n\}$, $y = \{y_1, y_2, \dots, y_n\}$. Let $L = \{l_i\}$ be a set of non-singular transformations of variables $x = l_i y$. The operators l_i are defined by functions $g_1^{(l)}, \dots, g_n^{(l)}$, where $x_k = g_k^{(l)}(y)$, $k = 1, 2, \dots, n$. A network realizing a function $f(x)$ can be obtained from the network for $h(x)$, if $f(x) = f(l_i y) = h(y)$.

Definition 1 *Functions $f(x)$, $h(x)$ are called* equivalent with respect to the set L *if there exists $l_i \in L$ such that $f(x) = h(l_i x)$.*

The task of simplification of the network for $f(x)$ can be formulated as

Find a transformation l_i mapping $f(x)$ into a function $h(l_i x)$ simplest to realize.

In the following, as L it is selected the set of all non-singular linear transformations, i.e., transformations for which $g_i^{(l)}$ are linear functions [8]. Such transformations can be represented in the form

$$x = Ay + \alpha, \tag{1}$$

where $A = \{a_{ik}\}$ is a square matrix, $\alpha = \{\alpha_i\}$ is a vector, a_{ik} and α_i are equal to 0 or 1, $det A = 1$ (with calculations modulo 2). L forms a group. Denote the transformation (1) by (A, α).

2. All functions can be split into equivalence classes with respect to L. We call the *weight of* $f(x)$ the set of points x where $f(x) = 1$.

Table 1: Values of $N^m(n)$ for $m = 0, \ldots, 16$.

	$N^m(n)$																	
n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	$N(n)$
3	1	1	1	1	2	1	1	1	1									32
4	1	1	1	1	2	2	3	3	4									10
5	1	1	1	1	2	2	4	5	8	9	15	16	23	24	30	30	38	382

The set of all functions of n variables with the weight m is called the *class* C_n^m.

Obviously, if $f \in C_n^m$, then $f(lx) \in C_n^m$. Denote by $N(n)$ the number of all classes of n-variable functions with respect to L, and by $N^m(n)$ the number of classes of all functions in C_n^m with respect to L. Calculations by using the method by Pólya ([3]-[7]) [1] produce values $N^m(n)$ and $N(n)$ for $n = 3, 4, 5$ shown in Table 1.

3. We will define a Boolean function by the set of vertices of n-dimensional unit cube or by a set of input vectors corresponding to these vertices

$$f(x) \sim \cup_{i=1}^{m} p_i, \qquad (2)$$

where $f(p_i) = 1$.

We arbitrarily select among p_i linearly independent vectors $p_1, p_2, \ldots, p_j$ such that remaining vectors can be expressed in terms of them as

$$(p_{j+1}, \ldots, p_m) = (P_1, \ldots, p_j)B,$$

where B is a $(j \times (m-j))$, $j \leq n$, matrix. When the set of vectors (2) is written as a matrix whose columns are p_l, then $f(x) \sim \phi \cup \phi B$, where ϕ is a matrix whose columns are vectors $p_1, \ldots, p_j$ and $\cup$ denotes the union of matrices [2]

Theorem 1 *Two functions $f(x)$ and $h(x)$ are equivalent with respect to the set of linear transformations of the form $(R, 0)$ iff they can be represented in the form $f(x) \sim \phi \cup \phi B$, $h(x) \sim \psi \cup \psi B$. In this case, R can be determined from the equation $R\psi = \phi$.*

[1] Pólya enumeration theorem (*Comment by the translator*)

[2] The union of two Boolean matrices A and B, with the same number of rows and columns, is the Boolean matrix whose element c_{ij} in row i and column j is the union of corresponding elements a_{ij} in A and b_{ij} in B. (*Comment by the translator*)

Corollary 1 *Two functions $f(x)$ and $h(x)$ are equivalent with respect to L iff for a vector α, $f(x) \sim \phi \cup \phi B$, it holds $h(x+\alpha) \sim \psi \cup \psi B$.*

Corollary 2 *If in some representations of $f(x)$ and $h(x)$, matrices ϕ and ψ have different number of columns, then $f(x)$ and $h(x)$ are non-equivalent.*

A function is called *minimal representant of a class of functions* if it can be realized in the simplest way by the given method.

Consider splitting into classes of all functions of the form $I \cup IC$, $I = \{a_{ik}\}$, $a_{ik} = 0$ for $i \neq k$, $a_{ii} = 1$). Select as $f(x)$ and $h(x)$ arbitrary representations $f(x) \sim \phi \cup \phi P$, $h(x) \sim \psi \cup \psi Q$. The equivalence of $f(x)$ and $h(x)$ is identical to the equivalence of $I \cup IP$ with $I \cup IQ$. In this way, by using the table in [2], defined are all equivalence classes for $n = 4$ and determined minimal representations for the synthesis method specified in [2]. In the considered case, complexity of networks is determined by the number of gates.

Denote by $\mu(n)$ the average complexity of networks for the minimal representations of all Boolean functions of n variables.

Theorem 2 *In electrical networks with vacuum tubes $\mu \lesssim 5.81$.*

4. For a simultaneous realization of q functions, efficiency of the method increases if all linear functions are realized. There are $3n$ inputs. At first $2n$ outputs the assignments $\tilde{x}_1, \ldots, \tilde{x}_n$ are applied as static voltages. At remaining n inputs, signal x_i is applied as a pulse at the moment i. By using rectifiers, from variables $x_1, \ldots, x_n$, we generate $x_i + x_j$, $x_i + x_j + x_k$, etc. (Fig. 1). Each sum is feed to a counter register (trigger). From the trigger, we get both the function and its complement.

The complete network S to realize all linear functions requires $2(2^n - n - 2)$ rectifiers and the same number of triggers. In this network, $g_1, g_2, \ldots, g_q$ are subnetworks for minimal representations of functions $f_1, f_2, \ldots, f_q$. The network works in n clocks (Fig. 3).

5. We will now describe a method to determine a transformation minimizing the network for an arbitrary n.

Represent the function in the form (2). [3] Assign to each row a *characteristic* - pair of numbers, where the first number is the number of zero values in the row, and the second number is the number of 1 values. We

[3] Field of cubes where $f(x) = 1$, or 1-field. (*Comment by the translator*)

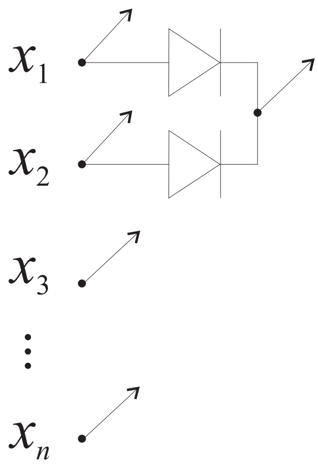

Figure 1: Network to generate linear functions.

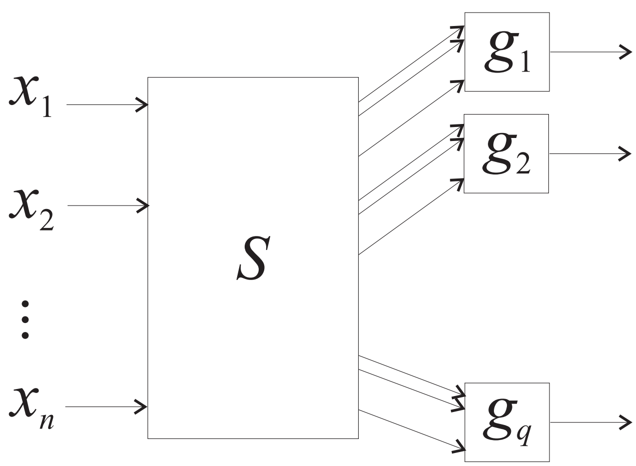

Figure 2: Structure of the network realizing n-variable functions.

say that the characteristic $[m_1, k_1]$ is *better* than the characteristic $[m_2, k_2]$ if $|m_1 - k_1| > |m_2 - k_2|$.

Multiply the table [4] from the left by the $(n \times n)$ matrix $T_1 = \{a_{ik}\}$, where $a_{ik} = 0$ for $i > k$, $a_{ii} = 1$. Select rows of the matrix from the top in the increasing values of characteristics. While selecting each row in the table (1-field) for $f(x)$, just a single row changes, corresponding to the selected row in the matrix. Continue the transformation by using a lower triangular matrix T_2 and convert some x_i into $\overline{x}_i$ such that in each row the zero values are dominant. In this way we get $f(T_2 T_1 x + \alpha) = \phi(x)$. In table $\pi(x)$ there are many zeros, and due to that the procedure on the average approaches the function vector of $f(x)$ to a zero vector, and $f(x)$ approaches a monotone symmetric function. The network is simplified at the price of appearance of invariants in ϕ' such that $\phi' \to \phi(x)$.

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24 VI 1958

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[7] Ashenhurst, R.L., ibid, page 293.

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[4] 1-field (*Comment by the translator*)

Remarks by the Editors

A translation of this paper by E.V. Nečiporuk has been published in *Automation Express*, April 1959, Division 2, *Network Theory and Telemetering*, 12-13.

The complete references used in the paper by E.V. Nečiporuk are

Povarov, G.N., "Synthesis of contact multipoles", *DAN*, 94, 6, 1954, 1075-1078.

Pólya, G., "Kombinatorische Anzahlbestimmungen für Gruppen, Graphen und chemische Verbindungen", *Acta Mathematica*, Vol. 68, 1937, 145254.

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Slepian, D., "On the number of symmetry types of Boolean functions of n variables", *Canad. J. Math.*, Vol. 5, 1953, 185-193.

Singer, T., "The theory of computing techniques", *Proc. ACM National Meeting*, Pittsburgh, Pennsylvania, USA, May 2, 1952, 287-291.

Ashenhurst, R.L., "The theory of counting techniques", *Proc. ACM National Meeting*, Pittsburgh, Pennsylvania, USA, May 2, 1952, 293-205.

Yablonskii, S.V., "Realization of a linear function in the class of series-parallel networks", *DAN*, 94, 5, 1954, 805-806.

DAN is an abbreviation for *Doklady Akademii Nauk SSSR*, *Reports of the Academy of Sciences of USSR.*

For reference [1]

A short abstract of the paper [1] taken from Belevitch, V., "Recent Russian Publications on Switching Theory", *IRE Transactions - Circuit Theory*, Reviews of Current Literature, March 1956, 78, is as follows.

(The paper) *Describes applications of Shannons method, where a switching* (p, q)*-network (with* p *inputs and* q *outputs) is realized as a tandem connection of two subnetworks, the first of which is disjunctive.*

Considers specially the case where the disjunctive network is a $(1, 2^n)$*-tree (requiring* $2^{n+1} - 2$ *contacts) and the second subnetwork is the universal* $(2^{2^n}, 1)$*-multipole, realizing all Boolean functions of* n *variables.*

It is proved that the universal multipole is realizable with

$$2\left(2^{2^n} - \sum_{i=0}^{n-1} 2^{2^i} + n - 2\right)$$

contacts.

It is proved that a set of k *arbitrary Boolean functions of* n *variables is realizable for any* m $(0 \leq m \leq, n)$ *by interconnecting* k *trees of* $n - m$ *variables with a universal multipole of* m *variables, the realization then requiring not more than*

$$2\left(k(2^{n-m} - 1) + 2^{2^m} - \sum_{i=0}^{m-1} 2^{2^i} + m - 2\right)$$

contacts.

The optimum choice of m *leads to an upper limit inferior to* $k2^{n+s}/(\log_2 k + n) - 2k$ *contacts if* $\log_2 k + n \geq 1$.

Analogous results are obtained for symmetric neworks, the trees being replaced by the lattices of elementary symmetric functions, and the universal multipole being limited to the realization of all symmetric functions.

In 1954, G.N. Povarov wrote his candidate for PhD degree thesis *Investigation of Contact Networks with Minimal Number of Contacts*, while working in the area of mathematical logic at the Institute for Control (Avtomatika i Telemekhanika) of the Academy of Sciences of SSSR. In 1965, he received the degree Doctor of Physical -Mathematical Sciences at the Moscow Engineering and Physical Institute, Moscow, Russia and has been appointed for a Professor at the Department of Cybernetics at this Institute.

For reference [8]

The name of Yablonskii, is also translated as Jablonsky.

A short abstract of his paper [8] is taken from Belevitch, V., "Recent Russian Publications on Switching Theory", *IRE Transactions - Circuit Theory*, Reviews of Current Literature, March 1956, 78.

(The paper) *Discusses the realization of the sum (mod* 2*) of* n *variables as a series-parallel switching 2-pole. In accordance with a result of Riordan, the number of contacts is smaller than* $9n^2/8$.

These abstract above as presented in the mentioned issue of *IRE Transactions - Circuits and Systems* are based on the abstracts in the section "Mathematical Theory of Electrical Circuits" of *Referativnyj Zhurnal (Matematika)*, usually abbreviated as (RZhM), in English *Reference Journal for Mathematics*, that has been published starting from 1954, and the Redactor of *RZhM* at that time was Prof. D.J. Panov. The author of English translation of abstract V. Belevitch was with Centre d'Études et d' Éxploitation des Calculateurs Electroniques, Brussels, Belgium.

ЭДУАРД ИВАНОВИЧ НЕЧИПОРУК

Figure 3: Eduard Ivanovič Nečiporuk.

Publications by Eduard Ivanovič Nečiporuk

The following list of publications by E.I. Nečipouk has been compiled in *Problemi Kibernetiki*, No. 26 , 1973, 10-17. The photo of Nečiporuk is reproduced from the same issue.

"Network synthesis by linear transformation of variables", *DAN SSSR*, Vol. 123, No. 4, 1958, 610-612, translation in English in *Automation Express*, Vol. 1, No. 8, 1959, 12-13.

"Transformations of contact-rectifier networks", *Vestnik Leningradskogo Universiteta*, Ser. mathem., Vol. 13, No. 3, 1959, 148.

"On multi-terminal networks realizing multiple-valued logic functions", *Problemi Kibernetiki*, Vol. 5, Fizmatgiz, Moscow, 1961, 49-60 translation in English *Automation Express*, Vol. 4, No. 4, 1962, 11-12.

"On complexity of superposition in bases containing non-trivial linear formulaes with zero cost", *DAN SSSR*, Vol. 137, No. 5, 1961, 560-563, translation in English *Automation Express*, Vol. 3, No. 10, 1961, 6-8, review in *Math. Rev.*, Vol. 29, 227.

"Synthesis of $\mathcal{R}$ networks", *DAN SSSR*, Vol. 137, No. 5, 1961, 1078-1081, Reviewed in *Math. Rev.*, Vol. 26, 231.

"Complexity of systems in certain bases containing non-trivial elements with zero cost", *DAN SSSR*, Vol. 139, No. 6, 1961, 1302-1303.

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"Rectifier networks", *Proceedings of International Symposium on Theory of Relay devices and Finite Automata*, Moscow, September 24 - October 2, 1962, 42-43.

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"Synthesis of rectifier networks", *Problemi Kibernetiki*, Fizmatgiz, Moscow, Vol. 9, 1963, 37-44, translation in English *Automation Express*, Vol. 6, No. 3, 1963, 38-39.

"Self-correcting rectifier networks", *DAN SSSR*, Vol. 156, No. 5, 1964, 1045-1048, translation in English *Automation Express*, Vol. 7, No. 2, 32-33. reviewed in *Math. Rev.*, Vol. 29, 630.

"Synthesis of logic networks in incomplete and singular bases", *DAN SSSR*, Vol. 152, No. 2, 299-301.

"Synthesis of networks with threshold elements", *DAN SSSR*, Vol. 154, No. 4, 1964, 763-766, reviewed in *Math. Rev.*, Vol. 28, 1127.

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"On topological principles of self-correcting", *DAN SSSR*, Vol. 179, No. 4, 1968, 786-78.

"Correcting of shortcuts in rectifier and contact networks", *Kibernetika*, Kiev, No. 5, 1968, 40-48.

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"Realization of disjunction and conjunctions in certain monotone bases", *Problemi Kibernetiki*, Nauka, Moscow, Vol. 23, 1970.

"Topological principles of self-correcting", Part 2, *Problemi Kibernetiki*, 1973, No. 26, 19-26.

Foreword

Ever increasing complexity of digital devices requires continuous development of new approaches or improving existing optimization methods for logic networks.

Linear transformation of variables is a classical method for optimization of logic networks. It consists in a particular permutation of elements in the function vector in order to group them in such a way that the permuted function f_σ has simpler realization than the initial function f. It is assumed that the permutation considered can be expressed as a linear combination of variables. Therefore, for an n-variable function, this linear transformation can be expressed by a $(n \times n)$ non-singular over $GF(2)$ matrix σ, where $GF(2)$ is the Galois field of order 2.

In the last decade, this method has been readdressed and advantageously exploited by several authors, see references enclosed. The main problem is to determine for a given function a suitable linear transform for particular applications intended. Several algorithms proposed for this task, can be classified as

1. Exact algorithms, which due to complexity of the problem, are restricted to functions of up to seven variables.

2. Deterministic algorithms, by using different mathematical operators as, for example, the autocorrelation functions to determine the linear transformation of variables. Solutions produced often depend on the selection of values of some parameters. Minimization of the implementation time of these algorithms can be achieved by restricting the number of possible values for the algorithm parameters at a price of slightly reduced quality of the solutions produced.

3. Heuristic algorithms, with no guarantee of the quality of the solutions, but ensuring efficient implementation in terms of both space and time.

The present issue of The Reprints from History reprints the paper by E.I. Nečiporuk from 1958 introducing the method of linear transformation of variables of Boolean functions in order to minimize their representations and subsequently realizations. This is the first publication where that approach to the optimization of logic networks has been proposed. The method leads to an algorithm that belongs to the second class of algorithms as enumerated above.

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