

A great disappearing act: the electronic analogue computer

Chris Bissell

The Open University
Milton Keynes, UK
c.c.bissell@open.ac.uk

Abstract

One historian of technology has called the analogue computer 'one of the great disappearing acts of the twentieth century'¹. This paper will look briefly at the origins, development and decline of the electronic analogue computer.

Analogue computers in the broadest sense have a long history. The slide rule, for example, can be thought of as an analogue computer. By the early part of the twentieth century various mechanical and electromechanical devices were used to perform complex calculations that could not be easily carried out by hand. But it was during WW2 and in the immediate aftermath that electronic components were developed that could be used to simulate accurately mathematical operations. In particular, the operational amplifier, whose origins lie in the pre-war efforts of Bell Labs to make possible continental telephony, revolutionised the field. Such amplifiers could realise, to a high degree of accuracy, the mathematical operation of integration, and thus the solution of linear differential equation models of dynamic systems.

During the 1950s and 1960s electronic and hybrid analogue computers were at the heart of modelling such technological systems as aerospace and industrial plant control. Heated debates took place about the relative virtues of analogue and digital computers. By the mid 1970s, however, analogue computers had largely disappeared. Yet, more recently, digital simulation tools have drawn much from the analogue computer approach. Whereas originally such analogue computers were designed to relieve engineers from the drudgery of computation, many of their features now form an integral part of the user interface of digital simulation tools.

The history of analogue computing thus has much to tell us about the way engineers have developed and exploited dynamic models of complex engineered systems. In addition to presenting aspects of the history of analogue computing, this paper will also explore the modelling implications of the analogue computer approach. Clearly, in a short paper only major features can be addressed. Those who wish to learn more are referred to the bibliography, particularly the publications of James Small and George Fox Lang.

Introduction: pre-electronic devices

Analogue computing devices, in the broadest sense, go back to ancient times: sundials or water clocks, for example. Another classic, the slide rule (in regular use by students and practising engineers, mathematicians and scientists until the 1970s) now seems to us to be almost as antiquated!

The 19th Century saw the invention of a number of mechanical analogue computing devices, mostly used to solve differential equations by means of mechanising the

¹ Paul Edwards, in a review of Small (2001)

process of integration. By the 1920s electrical network analyzers had been introduced, in which electrical analogues (networks of resistive and reactive components) were used to simulate larger-scale electrical networks and also other engineering problems involving partial differential equation models.

More or less contemporary with the first network analyzers was the development of electro-mechanical differential analyzers, the pioneering work being carried out by Vannevar Bush at MIT in the late 1920s and early 1930s. One of the most important parts of this work was Harold Hazen's development of precise servomechanisms, a contribution that was also important in the general theory of feedback (Bennett, 1993).

Bush's differential analyzer (Figure 1) used six wheel-and-disc integrators and so could solve two, third-order differential equations simultaneously. Data was entered by tracing appropriate input curves, and output was plotted automatically.

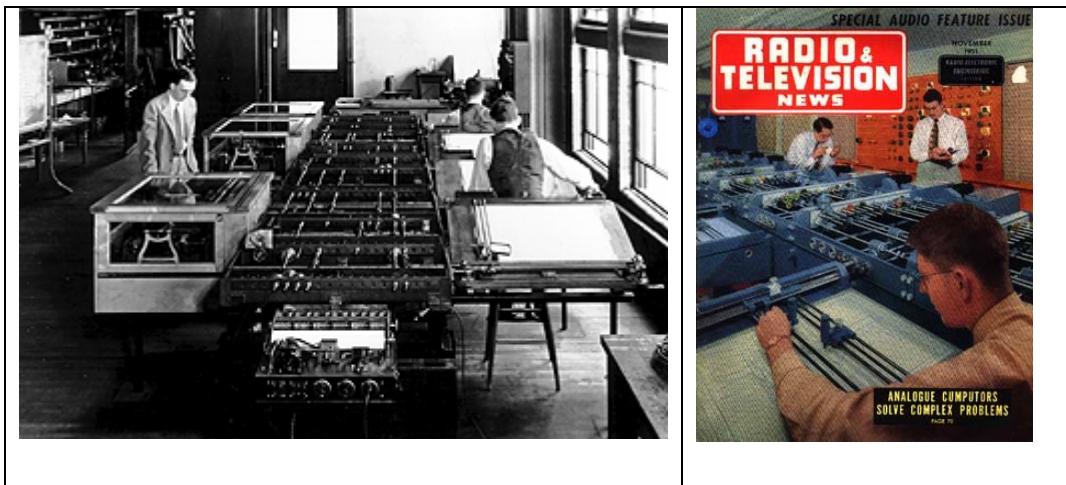


Figure 1: (left) The original MIT differential analyzer, image courtesy MIT Museum; and (right) the popularisation of the use of such a device (General Electric Differential Analyzer) as late as 1951

During the 1930s and early 1940s similar machines were developed in a number of countries, including the major combatants of WW2. These analyzers were of great importance for military applications. For example, a machine developed at Manchester University in the UK, based on the MIT design, was used for such applications as radar propagation, the development of the magnetron, and the computation of projectile trajectories (Small, 2001, pp 48-9).

WW2 also saw the development and widespread application of electronic systems, especially for fire control (gun aiming) and other feedback control applications (Bennett, 1993; Mindell, 2002). The key technology here was the operational amplifier, as discussed in the following section; most subsequent analogue computers depended to a large extent on the exploitation of such operational amplifiers. However, it is worth noting that at least one electrical, rather than electronic, device – Edwin Harder's *Anacom* – was still in operation (at Asea Brown Boveri in East Pittsburgh) in 1991! (Asprey, 1993)

The Electronic Analogue Computer

It is tempting to assume that the substitution of one technology for another is a simple matter of technical superiority. Indeed, claims of exactly this type have been made to explain why electronic analogue computers soon replaced mechanical differential analyzers; and why digital computers quickly replaced analogue ones. All too often such claims are simplistic and even downright wrong (it was noted above that the Anacom was still in operation in 1991, for example).

The key to the electronic analogue computer was the operational amplifier: a stable, linear device developed initially for use in repeaters in US transcontinental telephony. Simple circuits using operational amplifiers can carry out, to a great degree of accuracy, the mathematical operations of integration, multiplication, subtraction, inversion, etc, and can thus be used to simulate systems modelled by differential equations. The other major component in analogue computers was the coefficient potentiometer, used to ensure the proper scaling of the electrical analogue variables.

However, as James Small remarks:

The new electronic analog devices were in several respects inferior [...] The early operational amplifier produced results far less precise than those from wheel and disk integrators. The mechanical machines [could] perform integration with respect to mathematical functions, whereas electronic analog computers could only perform calculus operations with respect to time.

(Small, 1993, p9)

In fact, the op-amp required considerable further development until it achieved its modern, almost ideal, characteristics. The reasons that the early electronic analogue computers rapidly gained ground include the more prosaic ones of smaller size, easier portability, lower cost, and higher speed. (We shall return to the last point later.) Another important factor was the way that electronic analogue computers rapidly became an inherent part of the research and development of the military, aerospace, and process engineering sectors, as indicated in the following table.

Some important developments²

Project/machine	developer	date(s)	Notes
Project Cyclone	US Navy / Reeves	1946	REAC developed; analogue / digital comparison of performance; public dissemination
Project Typhoon	US Navy / RCA	1947	Single, purpose-built, machine; improved DC op-amp
MIT Flight Simulator	US Navy / MIT	1948-58	
RAND analog computer	RAND Corporation	1948	REAC modification leading to TRAC
GEDA/BEAC	US Air Force	c1950	Collaboration with Goodyear & Boeing; increased funding for ballistic missile programme
TRIDAC	RAE / Elliot Bros	1950-1955	Missile simulation
LACE	English Electric	1953-6	General purpose, but missile simulation again a major application

² Acronyms are listed at the end of this paper.

The use of an analogue computer involved the interconnection of components such as operational amplifiers so as to simulate the system under investigation. Figure 2 shows a small analogue computer with such connections in place.

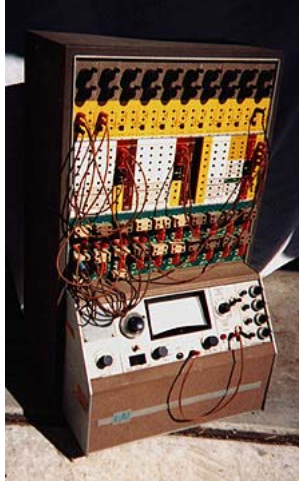


Figure 2: A small analogue computer wired up for simulation (Electronic Associates)

A large part of the skill of using an analogue computer was to determine the optimum configuration of components for a given problem. For example, an 'obvious' solution to a problem modelled by a linear differential equation with constant coefficients is simply to use enough integrators, inverters and potentiometers in a chain to 'undo' the effects of the various derivative terms. But this is an inefficient solution, and can also give rise to problems of accuracy and limited frequency response. A better solution is often to design a configuration involving one or more feedback loops, in order to reduce the number of op amps required (at the expense of eliminating direct access to some intermediate variables). This is not the place to go into the details of this 'art'; interested readers are referred to Lang (2000).

The major driver in post-war analogue computing was the aerospace industry, but such computers were also widely used in, for example, nuclear engineering, process control, and medical applications.

The Analogue-Digital Debate

I have hinted above that the commonly accepted view that analogue computers had a short flowering, and soon fell into decline with the advent of the digital computer, is a highly inaccurate historical reading. Readers are again referred to Small (2001, Chapter 8) for a full discussion. But it is worth noting some of the major features of the debate here. Perhaps the most important thing to appreciate is that all the keywords are complex, contested and of negotiated meaning.

Speed. Digital computers could carry out repeated mathematical operations at very high speed; yet analogue computers (which did not carry out mathematical operations *per se*) could be much faster for a certain class of simulation. For example:

At Project Cyclone digital computers were [...] used to verify the accuracy of results from the analog computer. [...] In the simulation of a guided missile in three

dimensions, the average runtime on the analog computer facility was approximately one minute. The check solution by numerical methods [...] took from 60 to 130 hours to solve the same problem. (Small, 1993, p. 11)

Accuracy and Precision. Analogue computers certainly produced less accurate numerical results for the solution of, say, a differential equation. Yet other, 'engineering' questions arose. How accurate is 'good enough' for a particular application? How good was the original model? And digital simulations also raised issues about numerical techniques.

A '*feel for engineering*'. One of the most common advantages claimed for analogue computers was the way the simulation was closely related to the engineering problem. Once the analogue computer was set up, it was comparatively simple to model changes in parameters, or uncertainties in modelling assumptions, simply by adjusting the appropriate physical variables in the analogue equipment.

Applications in Control Engineering

One of the early important applications of electronic analogue simulation for engineering design was George A Philbrick's Automatic Control Analyzer (ACA), developed between 1938 and 1940 (Holst, 1982). Philbrick was an engineer at the Foxboro Instrument Company. He saw the ACA as not only a tool for solving a mathematical problem, but also a powerful visualisation device (more of this later):

Demonstration of the dynamic characteristics of processes, hitherto the work of mathematicians, can be made by anyone who is able to follow a flow diagram. The effect of varying amounts of reset or control, for example, is shown in a lively graphic form which is better than any amount of wordy description. Every phenomenon of Automatic Control may be pictured at will, and with no more effort than is needed to turn knobs.

(Philbrick, memo of 1940, cited in Small, 2001, p231)

One of the important areas of postwar development was the rapid strides made in control theory. The major features of what we now refer to as classical control were developed during and just after WW2. In the immediate postwar period, a great deal of attention was devoted to the analysis of non-linear feedback systems, which were difficult to model analytically. A great advantage of analogue computing was the way that nonlinearities could be included in the simulation (although pure time delay was problematic). Derek Atherton (2002) recalls how linearised models could be solved analytically, and nonlinearities then included in an analogue simulation, before deciding upon a final controller design. In the 1950s the Manchester University analogue computer was widely used for control problems. (Manchester University was one of the three in the UK with specialist control groups – Imperial College, London and Birmingham University were the two others.) The Manchester machine included nonlinear functions to model hysteresis, backlash, deadzone, saturation and relays. Time delays were included by means of Padé approximations or magnetic tape delay.

Modelling, Computing, and Visualization

As should be clear from the above, the development of analogue computers went hand in hand with ideas about engineering modelling, and in particular the way mathematics was used by (predominantly electrical/electronic) engineers in the first decades of the 20th Century. See Puchta (1996), Kline (2000) and Bissell (in print) for a further discussion of this theme.

Notions of ‘the art of engineering’ and ‘intuition’ and ‘feel’ for good engineering design informed the whole analogue computing project. Small (2001) again:

[...] the use of electronic analogue computing led not only to the growth of tacit forms of technological knowledge but also to the revision of engineering theory. They enabled engineers to build active models that embodied and operationalised the mathematical symbolism of engineering theory. (p. 274)

Recent years have seen a return to this approach. Modern interfaces to digital computer simulation packages owe much to the analogue computing tradition. In Matlab/Simulink™, for example, users interconnect functional blocks on-screen in much the same way that earlier engineers used analogue computers. The ‘feel’ that earlier engineers gained from such systems are now reflected in the ‘feel’ that contemporary engineers gain from packages such as Matlab™, illustrated in Figure 3. Note the use of symbols derived from earlier analogue computer use, and the oscilloscope metaphor.

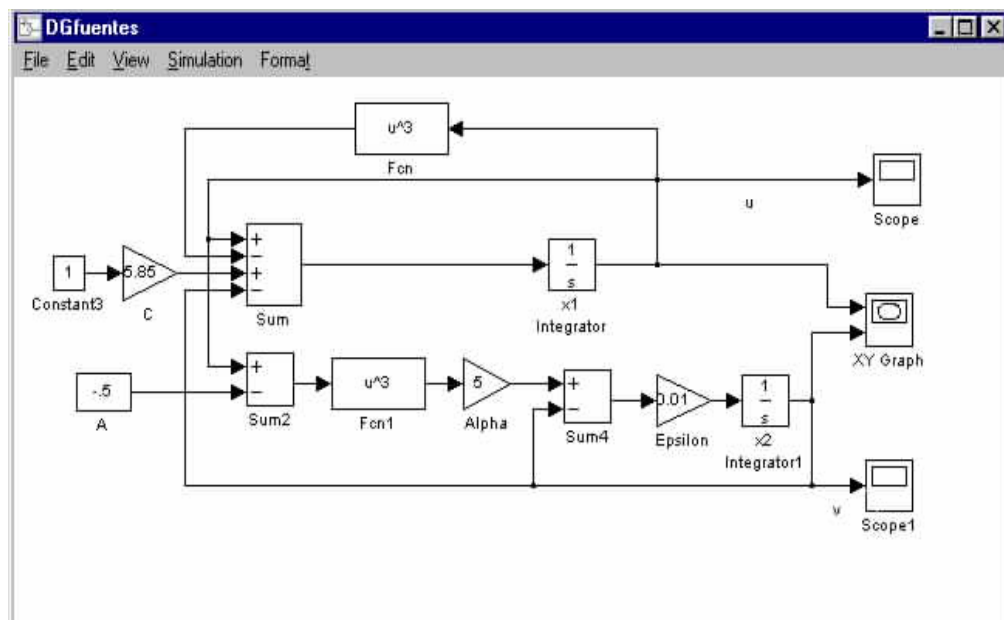


Figure 3: A Simulink™ display (Taubert, 2002)

Such visualisation is a powerful tool. To conclude, let me mention briefly one non-electronic analogue computer from the early 1950s. The Phillips machine was a hydraulic simulator of a national economy. It wasn't particularly accurate by modern standards (about 5%) – but, as mentioned above in the context of electronic analogue computers, accuracy was not necessarily the prime concern. In the Phillips machine monetary flows (income, tax, even foreign exchanges) were represented by flows of coloured water. Up to seventh-order systems could be simulated.

As Morgan and Boumans (2004) remark:

Seeing the machine working is different from pictures of it, as those who have seen the machine working³ readily attest. Spectators could not only see the red water streaming through the pipes, but also hear the bubbling and splashing as it ran through the machine. They were able to see not a 2-D picture or system of equations, or even a static 3-D representation, but the kind of interrelated and dynamic cause-effect changes over time that economists suppose to happen in the circular flow of the aggregate economy. The working machine was a 4-D representation. (p. 390)

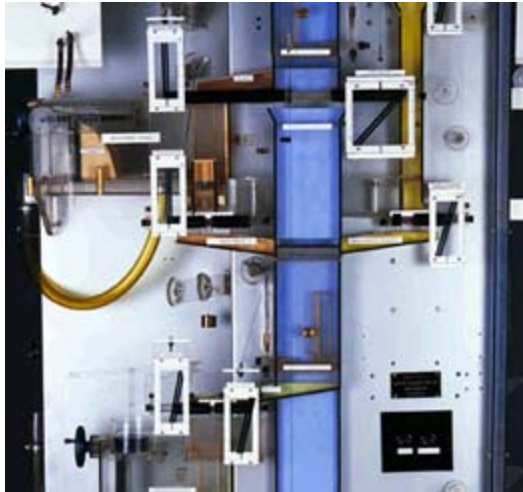


Figure 4: The Phillips hydraulic simulator of a national economy

It is no accident that Phillips was an electrical engineer. His economic simulator was in the grand tradition of analogue computers developed in that discipline, as are the current digital software packages that are used to simulate complex technological devices and systems from all domains of engineering.

All computer simulations are closely bound with users' conceptualisations of the model being used. One of the striking features of all analogue computers, but particularly electronic ones, is the way users developed a very close – and sometimes intuitive – relationship with the machine and its configuration. As Lang (2000) remarks:

Learning to use an analog computer forced the practitioner to cross the (unnecessary) boundaries between engineering disciplines; this may be its greatest virtue. [...] There is no better way to develop a "gut feel" for the interplay between physics and mathematics than to experience such an interaction. The analog computer was a powerful interdisciplinary teaching tool; its obsolescence is mourned by many educators in a variety of fields. (p. 23)

Decline

With a few exceptions, analogue computers had fallen out of use by the mid 1970s, but there is no one, single, cause for the overwhelming shift to digital. We have already discussed some of the arguments about speed of simulation, and 'intuitive feel'; but as digital processor power increased, and digital machines became more user-friendly, the 'advantages' of analogue became increasingly difficult to defend.

³ Including the author

As a general purpose device, too, digital computers had a much greater market, particularly in business applications.

Nevertheless, at a time when many teachers of engineering are bewailing the lack of mathematical expertise in their students, and their poor understanding of mathematical models of engineered systems, it may be that we have lost something important with the passing of the analogue computer.

Larry Owens (1986) concludes, in his interesting paper on Bush's differential analyzer, that

the analyzers were exercises in the language of early 20th century engineering. Forged in the machine shop, the analyzers spoke the Graphic Language while they drew profiles through the landscape of mathematics. The student could find in these machine texts a catalog of his technical universe, lessons in the nature of mathematics and its instruments, and even expressions of the ethos which pervaded engineering education... When engineers and their new corporate and federal supporters turned to the problems of computation at the end of the Second World War, they discovered the need for new texts in a more modern idiom, composed by a younger generation of inventive authors. (p95)

Owens' proposition is that machines are 'texts' and embody language and values of their time. (See also Puchta, 1996, for a related discussion.) While Owens' argument is developed in the specific context of the differential analyzer, similar points can surely be made about later machines. And an intriguing question is this: if a particular machine embodies particular language and values, which can help in engineering understanding, does this specific embodiment at the same time obscure other potentially helpful languages and values?

Acronymns

BEAC	Boeing Electronic Analog Computer
GEDA	Goodyear Electronic Differential Analyzer
LACE	Luton Analogue Computing Engine
MIT	Massachusetts Institute of Technology
RAE	(UK) Royal Aircraft Establishment
RAND	Initially established as R&D division of Douglas Aircraft Corporation
RCA	Radio Corporation of America
REAC	Reeves Electronic Analog Computer
TRAC	The Rand Analog Computer
TRIDAC	Three-Dimensional Analogue Computer

References

- Aspray, W. (1993), Edwin L. Harder and the Anacom: Analog Computing at Westinghouse, *IEEE Annals of the History of Computing*, Vol. 15, No. 2, pp. 35-52
- Atherton, D. P. (2002), The Role of Analogue Computers in Classical Control Design. In: *Procs. IFAC 15th Triennial World Congress*, Barcelona, Spain.
- Bennett, S. (1993), *A History of Control Engineering 1930-1955*, Peter Peregrinus: Stevenage
- Bissell, C. C. (in print), Models and Black Boxes: mathematics as an enabling technology in the history of communications and control engineering. *Revue d'Histoire des Sciences*

- Holst, P. A. (1982), George A. Philbrick and Polyphemus – The First Electronic Training Simulator, *Annals of the History of Computing*, Vo. 4, No. 2, April
- Kline, R. (1994), World War II: a Watershed in Electrical Education, *IEEE Technology and Society Magazine*, Summer, pp. 17-23
- Kline, R. (2000), The Paradox of Engineering Science, *IEEE Technology and Society Magazine*, Fall, pp. 19-25
- Lang, G. F. (2000), Analog was not a Computer Trademark!, *Noise and Vibration Control*, August, pp. 16-24.
- Mindell, D. A. (2002), *Between Human and Machine*, Johns Hopkins University Press: Baltimore / London
- Morgan, M. S. and Boumans, M. (2004), Secrets Hidden by Two-Dimensionality: the Economy as a hydraulic Machine. In: De Chadarevian, S. and Hopwood, N. (eds), *Models: The Third Dimension of Science*, Stanford University Press
- Owens, L. (1986), Vannevar Bush and the Differential Analyzer: The Text and Context of an Early Computer, *Technology and Culture*, Vol. 27, No. 1, pp. 63-95
- Puchta, S. (1996), On the Role of Mathematics and Mathematical Knowledge in the Invention of Vannevar Bush's Early Analog Computers, *IEEE Annals of the History of Computing*, Vol. 18, No. 4, pp. 49-59
- Small, J. S. (1993), General-Purpose Electronic Analog Computing, *IEEE Annals of the History of Computing*, Vol. 15, No. 2, pp. 8-18
- Small, J. S. (2001), *The Analogue Alternative*, Routledge: London/New York
- Taubert, K. (2002) <http://www.math.uni-hamburg.de/home/taubert/simulink.pdf>

Websites (all accessed 24 June 2004)

- <http://dcoward.best.vwh.net/analog/> : *Analog Computer Museum* (source of Figs. 1b, 2)
- <http://www.taswegian.com/SRTP/javaslide/javaslide.html> : *Slide rule simulation!*
- <http://web.mit.edu/mindell/www/analyzer.htm> : *Page on Bush's analyzer*