Temperature Coefficients of Elastic Constants of Quartz

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We have reported in the paper (JIEE Japan, Vol. 54, No. 543, p.940, Oct. 10, 1933 on October 1933) on the thickness vibration of quartz oscillating plates cut in parallel to X-axis (Figure 1) that the vibration mode is sharing vibration and frequency is expressed as

\[ f = \frac{q}{2a} \sqrt{\frac{c}{\rho}} \quad c = \frac{1}{2} (c_{11} - c_{12}) \sin^2 \theta + c_{44} \cos^2 \theta - c_{14} \sin 2\theta \quad \cdots \quad (1) \]

Based on the equation (1), we estimated that temperature coefficients of oscillation would change along with the change of \( \theta \) and we reported the results of the measurement of temperature coefficients using such plates. Furthermore, we informed that temperature coefficients of adiabatic elastic constants of \( c_{44}, c_{14} \) etc. would be calculated and that these results would be presented later. In this paper, we will discuss on this issue.

Variation of frequency \( f \) in equation (1) by temperature is given as

\[ 2 \frac{1}{f} \frac{\partial f}{\partial T} = \frac{1}{c} \frac{\partial c}{\partial T} - \frac{1}{\rho} \frac{\partial \rho}{\partial T} = \frac{2}{a} \frac{1}{\rho} \frac{\partial \rho}{\partial T} \quad \cdots \quad (2) \]

Line expansion coefficients of line of length \( a \) with the arbitrary direction (direction cosine \( l, m, \) and \( n \)) having density \( \rho \) is given as

\[ \frac{1}{\rho} \frac{\partial \rho}{\partial T} = \frac{1}{x} \frac{\partial x}{\partial T} + \frac{1}{y} \frac{\partial y}{\partial T} + \frac{1}{z} \frac{\partial z}{\partial T} \quad \cdots \quad (3) \]

\[ \frac{1}{a} \frac{\partial a}{\partial T} = \rho \frac{1}{x} \frac{\partial x}{\partial T} + m^2 \frac{1}{y} \frac{\partial y}{\partial T} + n^2 \frac{1}{z} \frac{\partial z}{\partial T} \quad \cdots \quad (4) \]

\[ \frac{1}{x} \frac{\partial x}{\partial T} = 13.7 \times 10^{-6}^\circ C \]

\[ \frac{1}{z} \frac{\partial z}{\partial T} = 7.5 \times 10^{-6}^\circ C \]

(by G. W. C. Kaye and T. H. Laby : "Physical and Chemical Constants", p.56)

Therefore, equation (2) of temperature coefficient becomes as

\[ 2 \frac{1}{f} \frac{\partial f}{\partial T} = \frac{1}{c} \frac{\partial c}{\partial T} + (7.5 + 12.4 \times \cos^2 \theta) \times 10^{-6} \quad \cdots \quad (6) \]
By measuring frequencies and temperature coefficients for plates cut at $\theta = 140^\circ$ (angle between X-axis and the plate face was within 0.5') and $\theta = 90^\circ$ (correspond to Y-cut and angle between X-axis and the plate face was within 0.5') are shown in Table 1 and Fig 2.

Temperature coefficients of Y-cut plate increase along with the decrease of the thickness of plates and reach the values listed in Table 1.

From Fig 2, at $\beta = 9^\circ 46'$ ($\theta = 137^\circ 59'$) and $\theta = 54^\circ 45'$ (This angle was reported in the paper published on October, 1933) temperature coefficients become zero.

We substitute these values and temperature coefficient $103 \times 10^{-6}/^\circ C$ at $\theta = 90^\circ$ into equation (6) as follows,

$$0 = \left( \frac{1}{c} \frac{\partial c}{\partial T} \right)_{137^\circ 59'} + \left( 7.5 + 12.4 \times \cos^2 137^\circ 59' \right) \times 10^{-6} \quad \text{---------------------- (7)}$$

$$0 = \left( \frac{1}{c} \frac{\partial c}{\partial T} \right)_{54^\circ 45'} + \left( 7.5 + 12.4 \times \cos^2 54^\circ 45' \right) \times 10^{-8} \quad \text{---------------------- (8)}$$

$$2 \times 103 \times 10^{-6} = \left( \frac{1}{c} \frac{\partial c}{\partial T} \right)_{90^\circ} + 7.5 \times 10^{-8} \quad \text{---------------------- (9)}$$

Then we use following values.

$$c_{96} = \frac{1}{2} (c_{11} - c_{12}) = \frac{1}{2} \left( 85.45 - 7.26 \right) \times 10^{10} \text{ dynes/cm}^2$$

$$c_{44} = 57.09 \times 10^{10} \text{ dynes/cm}^2$$

$$c_{14} = -16.87 \times 10^{10} \text{ dynes/cm}^2$$

From equations (9), (1), and (10),

$$\left( \frac{1}{c} \frac{\partial c}{\partial T} \right)_{90^\circ} = \frac{1}{c_{96}} \frac{\partial c_{96}}{\partial T} = +199 \times 10^{-6}$$

$$\frac{\partial c_{96}}{\partial T} = +77.8 \times 10^6 \quad \text{---------------------- (10)}$$

It is obvious that

$$\frac{\partial c}{\partial T} = \sin^2 \theta \frac{\partial c_{96}}{\partial T} + \cos^2 \theta \frac{\partial c_{14}}{\partial T} + 2 \sin \theta \frac{\partial c_{14}}{\partial T} \quad \text{---------------------- (12)}$$

From equations (7), (8), (10), (11) and (12)

$$\frac{\partial c_{14}}{\partial T} = -113.5 \times 10^6, \quad \frac{1}{c_{44}} \frac{\partial c_{14}}{\partial T} = -199 \times 10^{-6} \quad \text{---------------------- (13)}$$

$$\frac{\partial c_{14}}{\partial T} = -18.5 \times 10^6, \quad \frac{1}{c_{14}} \frac{\partial c_{14}}{\partial T} = +110 \times 10^{-6} \quad \text{---------------------- (14)}$$
Once we obtain temperature coefficients of adiabatic eristic constants, we can derive temperature coefficients of frequency using equations (1), (6), (11), (13) and (14). Figure 3 indicates the results of the calculation. Several small circles around the curve in Figure 3 correspond to the measured values of temperature coefficients which we reported in the previous paper on October 1933. The curve in Figure 3 shows good agreement with the experimental data.

Additionally, Temperature coefficient $C_{11}$ in equation $C_{60} = \frac{1}{2} (C_{11} - C_{12})$ can be calculated by measuring temperature coefficient of frequency of X-cut plate. Then we can get temperature coefficient of $C_{12}$. We will report these in the next time.

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